# Bubbles against Financial Repression

by Guillaume Plantin*

Abstract. During a financial crisis, a central bank temporarily subsidizes the interest rate so as to maintain borrowing at normal levels. Savers may search for yield and blow rational stochastic bubbles that generate a higher expected return than the policy rate before bursting at the end of monetary easing. Unlike standard rational bubbles, that are not monetary phenomena, these bubbles are "bad" in the sense that they crowd out investments that would otherwise generate a higher expected return than that on the bubbles.

## Introduction

An increasing number of observers contend that the low monetary-policy rates that have prevailed in advanced economies since 2008 have had the unintended consequences of blowing bubbles on risky assets instead of spurring much needed real investment. To be sure, the term "bubble" is used liberally in the public debate, and this point remains highly contentious. Yet the claim that low official rates have blown undesirable asset bubbles has undoubtedly gained significant traction since 2008, and even more so since the Covid-19 crisis.[1]

This claim flies in the face of baseline economic theory. First, it is the expectation of forever low real returns (possibly due in turn to permanent financial frictions) that induces (rational) bubbles. Monetary policy alone, on the other hand, cannot shift the real rate away from its flexible-price value forever, and so this "natural" real rate must be low for bubbles to arise. But then in this case bubbles are not a monetary phenomenon.

*Sciences Po, 28 rue des Saints-Peres, 75007 Paris, France (guillaume.plantin@sciencespo.fr).

[1]Examples abound, for example, in the blogosphere: "The Fed Has Created A Monster Bubble It Can No Longer Control," (https://seekingalpha.com/article/4324870-fed-created-monster-bubble-can-no-longer-control), "Barclays CEO warns of asset bubbles due to low interest rates", (https://www.cnbc.com/2019/08/01/barclays-ceo-warns-of-asset-bubbles-due-to-low-interest-rates.html), "The Fed Is Creating A Monster Bubble,"(https://www.forbes.com/sites/johnmauldin/2020/01/03/the-fed-is-creating-a-monster-bubble/#6aee77d72670), "Fed Trying To Inflate A 4th Bubble To Fix The Third,"(https://seekingalpha.com/article/4334541-fed-trying-to-inflate-4th-bubble-to-fix-third),"The Fed's corporate bond buying is stoking bubble fears,"(https://www.cnbc.com/2020/06/22/the-feds-corporate-bond-buying-is-stoking-bubble-fears.html),...

Second, whereas it is true that bubbles crowd out some investment, it is typically desirable because it enables to reallocate resources towards better purposes in situations of dynamic efficiency (Diamond, 1965; Tirole, 1985) or in the presence of financial imperfections (Farhi and Tirole, 2012a; Martin and Ventura, 2012; Miao and Wang, 2018; Woodford, 1990).

In sum, there is little theoretical support for the possibility that temporarily low policy rates suffice to blow asset bubbles, nor for the claim that trading such bubbles crowds out more profitable investments.

The goal of this paper is to offer a model in which rational asset bubbles may actually arise as an unintended and undesirable pure monetary phenomenon. We study an economy in which the central bank temporarily distorts the real rate below its natural value so as to subsidize borrowers during a financial crisis, modelled as an episode during which particularly severe financial frictions plague the economy for some finite but random time. We show that this may lead to temporary (stochastic) rational bubbles that burst at the end of this episode of monetary easing. These bubbles are undesirable in the sense that they evict the investments that low rates are meant to spur in the first place, despite earning a lower return than the total return on these investments.

These bubbles can be viewed as savers' response to financial repression. Their search for yield allows for the rise of bubbly assets that earn a higher return than the one artificially kept down as a well-intended policy. By doing so, savers hurt not only borrowers but also possibly themselves, as they divert resources away from high-return investments. The gist of our argument is quite simple when exposed in the following elementary model of financial repression. In this (toy) real model, unlike in the body of the paper, for simplicity, financial repression is a permanent tax on real storage that can create permanent deterministic bubbles.

## Gist of the argument

Consider an overlapping-generations economy in which a mass 2 of agents are born at each date, and live for two dates. They value consumption only when old, at which time they are risk-neutral. All agents can avail themselves of a linear storage technology with gross return $\rho > 1$. Half the population is comprised of savers who are endowed at birth with a consumption unit. The other half are borrowers, endowed at birth with an investment technology that transforms $I$ consumption units into $2\sqrt{I}$ units at the next date.

*Laissez-faire.* The equilibrium in this economy is such that savers lend $I^* = 1/\rho^2$ to borrowers at the rate $\rho$, and store $1 - I^*$ using the linear technology. The marginal return is equal to $\rho$ on both investments. Notice that a "natural rate" $\rho$ strictly above one precludes the occurence of bubbles.

*Laissez-faire with financial imperfections.* Suppose now that borrowers can only pledge a fraction

$\lambda \in (0, 1/2)$ of their output.[2] They are thus financially constrained, and borrow only $\hat{I} = 4\lambda^2/\rho^2 < I^*$ from savers at the prevailing rate $\rho$.[3] The marginal return on borrowers' technology, $\rho/2\lambda$, exceeds $\rho$.

*Financial repression.* Suppose that in the face of this financial friction, a planner seeks to restore the investment level to $I^* > \hat{I}$ using "financial repression": a tax on the linear storage whose proceeds are lump-sum rebated to savers.[4] This is a subsidy to borrowers' cost of capital.[5] A proportional tax $\tau = 1 - 2\lambda$ on the proceeds from linear storage does the job since optimal investment then solves:

$$2\lambda\sqrt{I} = \rho(1-\tau)I = 2\lambda\rho I, \tag{1}$$

implying

$$I = I^*. \tag{2}$$

*Bubbles against financial repression.* Suppose that $1 < \rho < 1/2\lambda$. The above tax on storage then implies that savers face an interest rate $2\lambda\rho < 1$ that can be conducive to bubbles, even though bubbles cannot exist under laissez-faire since $\rho > 1$. Consider for example the situation in which a bubble $B$ issued by the old savers at date 0 commands the maximum feasible return on a bubble—the growth rate of savings equal to one. In this case savers do not use the linear storage that has an inferior (after-tax) return. They lend $\underline{I} = 4\lambda^2 < I^*$ to borrowers (from $2\lambda\sqrt{I} = I$), and invest the residual in the constant bubble $B = 1 - \underline{I}$.

This bubbly equilibrium compares to non-bubbly financial repression as follows:

1. Bubbles in the presence of financial repression make borrowers worse off since they lead to investment $\underline{I} < I^*$ at a cost of capital $1 > (1-\tau)\rho$;

2. If $\rho(\rho - 1) > 1 - 2\lambda$, all cohorts of savers but the initial old savers issuing the bubble at date 0 are worse off than under non-bubbly financial repression because they earn a unit return on the bubble strictly smaller than the total (cum rebate) return $\rho$ on the storage that they neglect.

It is interesting to compare these results to that in a situation in which bubbles result from dynamic inefficiency. Suppose that $\rho < 1$ and that savers have an endowment strictly larger than

[2] As is well-known, such limited pledgeability may for example stem from a moral-hazard problem, whereby borrowers can divert and secretly consume their output but destroy a fraction $1 - \lambda$ of it by doing so.

[3] The average pledgeable return $2\lambda\sqrt{I}/I = 2\lambda/\sqrt{I}$ is smaller than the marginal return $1/\sqrt{I}$ and so borrowers invest until the former equals the prevailing rate $\rho$.

[4] Farhi and Tirole (2012b) model monetary easing in this very same way.

[5] Restoring investment to $I^*$ is for example what the planner would do if she was maximizing a social welfare function equal to the present value of aggregate consumption at each date discounted at $\rho$.

$1/\rho^2$. Suppose there are no financial frictions. In this case, a bubble can arise that raises the marginal return on investments to one from $\rho$. This negatively affects all cohorts of borrowers by raising their cost of capital. Yet all cohorts of savers benefit from this higher return. By contrast, our "bad" bubbles induced by financial repression hurt all agents except the initial issuers of the bubble.

In sum, we exhibit "bad" rational bubbles that are an inefficient response to financial repression in the sense that they crowd out investments with a higher return. This contrasts with the typically "good" rational bubbles that crowd out "bad" investments in situations of dynamic inefficiency or financial imperfections.

The goal of this paper is to show that such bubbles as a response to financial repression can arise as a pure monetary phenomenon when financial repression stems from the monetary authority temporarily imposing low interest rates aiming at mitigating financial imperfections. Going there from the above toy model raises a number of obvious issues. First, since deterministic bubbles as a monetary phenomenon would imply an impossible perpetual downward distortion of the real rate by monetary policy, bubbles can only be stochastic and burst when financial repression ends in our setup, a phenomenon reminiscent of the 2013 "taper tantrum".[6]

Second, it is important in the above model that real storage precludes negative storage. In the workhorse new Keynesian model, all private agents can by contrast seamlessly borrow and lend at the prevailing policy rate. No asset can earn a (risk-adjusted) one-period return above that implied by monetary policy in this case. Accordingly, the baseline new Keynesian setup needs to be augmented to reflect that, more realistically, not all tranches from all assets can be pledged against bank or central-bank loans in practice. We do so by introducing sophisticated investors who, as in Holmstrom and Tirole (1997), need to have incentive-compatible stakes in the investments that the central bank seeks to spur.

The paper is organized as follows. Section I presents the basic ingredients of our model. Section II outlines our main result. Section III discusses it and concludes

## Related Literature

This paper belongs to the literature that studies interest-rate policies as a tool to boost collateral values in order to mitigate financial-market imperfections (Benmelech and Bergman, 2012; Caballero and Simsek, 2020; Diamond and Rajan, 2012; Farhi and Tirole, 2012b). This literature has emphasized how subsidizing the interest rate this way may backfire into various forms of excessive risk taking. To our knowledge, we are the first to show that such excessive risk taking may

[6] Asset markets were ultimately not significantly affected during this episode after an initial small panic because the Federal Reserve quickly clarified its tapering plans.

materialize into "bad" rational bubbles that crowd out the wrong kind of investment.

Gali (2014) and Allen et al. (2018) have developed very tractable models to study the interplay of asset bubbles and monetary policy from which this paper heavily borrows. Bubbles are not a monetary phenomenon in Gali (2014). In Allen et al. (2018) the bubbles that are undesirable stem from excessive risk shifting by leveraged agents and are not pure unbacked assets as is the case here.

Our model also builds on the two-tiered moral-hazard model of financial intermediation by Holmstrom and Tirole (1997), that Meh and Moran (2010) have adapted to a monetary economy. In Holmstrom and Tirole (1997), sophisticated capital is more expensive than the unsophisticated one because of exogenous monitoring costs whereas it is an endogenous consequence of the privileged access of sophisticated investors to endogenous risky bubbles in our model.

Finally, it is interesting to relate this paper to the intermediary asset pricing literature pioneered by He and Krishnamurthy (2012, 2013). In this literature, negative shocks to sophisticated investors' wealth negatively affects all asset prices. It is the ability of these sophisticated investors to play around with risky, high-return bubbles that negatively affects overall investment in our setup.

# I. Baseline model

Time is discrete and indexed by $t \in \mathbb{N}$. The economy is populated by private agents, savers and borrowers, and by a monetary authority. Savers are in turn of two types, households and financiers. All agents use the same currency as unit of account ("cashless economy").

**Households.** A unit mass of households are born at each date, and live for two dates. Households consume a final good that they produce out of a continuum of intermediate goods indexed by $i \in [0, 1]$ using the technology

(3) $$C_t = \left( \int_0^1 C_{i,t}^{1-\frac{1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}},$$

where $\epsilon > 1$.

Each household $i \in [0, 1]$ owns when young a firm that transforms $L$ units of labor into $\alpha L$ units of good $i$, where $\alpha > 0$. Firms vanish after one production cycle. Households supply labor to firms when young. They rank bundles of consumption when young and old and labor $(C_Y, C_O, L)$ according to the criterion

(4) $$u(C_Y) + \beta C_O - \gamma L,$$

where $\beta, \gamma > 0$, and $u'$ is a decreasing bijection over $(0, +\infty)$.

**Financiers.** At each date, a unit mass of financiers are born and live for two dates. They consume the same final good as households but do so only when old, at which time they are risk neutral. Financiers are endowed at birth with $e > 0$ units of the final good.

**Entrepreneurs.** At each date, a unit mass of entrepreneurs are born and live for two dates. Like financiers, they consume the final good only when old, at which time they are risk neutral. Entrepreneurs born at date $t$ are endowed with a technology that transforms $x$ consumption units at date $t$ into $f(x)$ units at date $t+1$, where $f$ satisfies the Inada conditions.

Let us define

$$r^* \equiv \frac{\gamma\epsilon}{\alpha\beta(\epsilon - 1)}. \tag{5}$$

We suppose that

$$r^* > 1. \tag{6}$$

**Monetary authority.** The monetary authority announces at each date $t$ the gross nominal interest rate $R_t$ at which it is willing to borrow from and lend to private agents between $t$ and $t+1$. It sets $R_t$ applying the rule

$$R_t = r_t \left(\frac{P_t}{P_{t-1}}\right)^{1+\phi}, \tag{7}$$

where $r_t$ is the real rate of interest implied by households' rate of intertemporal substitution, $\phi > 0$, and $P_t$ is the date-$t$ price of the consumption good. We take $P_{-1} > 0$ as exogenously given.

**Frictions.** Two types of frictions plague this economy. First, all future consumption cannot be pledged in the financial market:

**Assumption 1.** ***(Financial frictions)*** *An entrepreneur investing $I$ generates the output $f(I)$ only if i) she has a stake larger than $\sigma f(I)$ in this output and ii) if a financier has a stake larger than $\mu f(I)$ in this output, where $\mu + \sigma < 1$. The investment generates no output otherwise.*

The appendix offers a simple microfoundation for this requirement that both the entrepreneur and a financier have sufficient "skin in the game". It borrows from the two-tier moral-hazard problem that Holmstrom and Tirole (1997) introduce in their model of investors with heterogeneous monitoring abilities. In a nutshell, entrepreneurs must have skin in the game in order to "behave" and avoid technologies that grant them large private benefits at the expense of output. Financiers have the ability to mitigate this moral-hazard issue by monitoring entrepreneurs. Monitoring comes at the cost of losing private benefits however, and so financiers must also have stakes in the output ensuring that they have proper incentives to monitor entrepreneurs. In the following we will state that "the incentive constraint is binding" if an equilibrium is such that entrepreneurs' stake is the minimum one and that "the monitoring constraint is binding" if so is financiers' stake.

The second friction is a nominal rigidity ensuring that monetary policy has real effects in the presence of uncertainty.[7] We simply assume that the firms that produce the intermediate goods set their prices in advance:

**Assumption 2.** ***(Prices are fixed in advance)*** *Intermediate-good firms operating at date $t$ set their prices at the outset of date $t$ using the information set available at $t-1$.*

## Perfect-foresight equilibrium

We define a perfect-foresight equilibrium as a situation in which agents optimize with correct expectations, markets clear, and inflation remains bounded.[8] We have:

**Proposition 1.** ***(Unique perfect-foresight equilibrium)*** *If $e$ is sufficiently small, there exists a unique perfect-foresight equilibrium. It admits a constant price level equal to $P_{-1}$. The real rate is $r^* > 1$. The real wage $w$, final good output $Y$, and entrepreneurs' investment $I$ solve*

$$w = \frac{\gamma}{\beta r^*} = \frac{\alpha(\epsilon - 1)}{\epsilon}, \tag{8}$$

$$u'(Y - I + e) = \beta r^*, \tag{9}$$

$$\begin{cases} (1-\sigma-\mu)f(I) = r^*(I-e) \text{ if both the incentive and monitoring constraints bind,} \\ (1-\sigma)f(I) = r^* I \text{ if only the incentive constraint binds,} \\ (1-\mu)f'(I) = r^* \text{ if only the monitoring constraint binds,} \\ f'(I) = r^* \text{ otherwise.} \end{cases} \tag{10}$$

**Proof.** See the appendix. ■

Positing that the exogenous endowment $e$ is sufficiently small *ceteris paribus* ensures that both types of savers, households and financiers, have strictly positive savings. Given the constant information set, that firms set prices in advance is immaterial, and the economy is identical to its flexible-price benchmark.

Optimal pricing by firms and the first-order conditions associated with households' labor supply and saving decisions yield $\{(8); (9)\}$, which characterizes the economy up to the amount $I$ that entrepreneurs invest. Condition (10) shows in turn how the severity of the financial frictions affect $I$. Binding incentive or/and monitoring constraints reduce $I$. The resulting smaller supply of

[7] The following Section II introduces uncertainty.

[8] This latter restriction is only meant to address the well-known criticism of the elusive terminal condition that applies to *any* model of inflation determination with the Taylor rule (Castillo-Martinez and Reis, 2019, e.g.).

storage available to households induces in turn a smaller output $Y$ from (9). The marginal return on investment $f'(I)$ is strictly above the real rate $r^*$ as soon as at least one constraint, either incentive or monitoring, binds, whereas financiers earn a higher return than $r^*$ only if the monitoring constraint binds.

Notice that this perfect-foresight equilibrium is unique because the economy is dynamically efficient ($r^* > 1$), and so there is no room for bubbles. This is of course not meant to describe the current environment. It rather serves to offer a clean benchmark in which bubbles *can only be a monetary phenomenon*.

A distinctive property of this model is that binding financial constraints negatively affect equilibrium quantities—firms' output and entrepreneurs' investment, whereas they leave the real interest rate $r^*$ unchanged. This feature of the model entirely owes to its special quasi-linear structure.[9] This sets an interesting benchmark in which a central bank merely aiming at replicating the constrained flexible-price outcome would track a constant real rate $r^*$ even when financial constraints bind. We now study how a central bank seeking instead to alleviate financial frictions by means of its interest-rate policy may unwittingly spur bad bubbles.

# II. Financial Repression

This section introduces a single departure from the baseline setup introduced in Section I. We suppose that the incentive-compatible stake of entrepreneurs that was a constant $\sigma$ now is a stochastic process. It starts out with the deterministic value $\hat{\sigma} \in (0, 1-\mu)$ at date -1. It remains at this value with probability $p \in [0, 1]$ at each subsequent date, or snaps to $\sigma < \hat{\sigma}$ with probability $1-p$, in which case it stays equal to this value $\sigma$ forever after. Notice that if $p = 1$ then the model from date 0 on is the perfect-foresight one of Section I with parameter $\hat{\sigma}$, whereas it is the same model with parameter $\sigma$ if $p = 0$. We have:

**Assumption 3.** ***(Temporary financial crisis)*** *If $p = 1$, the equilibrium is that in Proposition 1 in the case in which only the incentive constraint binds (the monitoring one does not). If $p = 0$, it corresponds to that in which no constraint binds.*

In other words, the economy starts out in a "financial crisis" modelled as a temporary situation during which a financial market failure reduces entrepreneurs' investment. After this crisis stochastically ends—when entrepreneurs' incentive-compatible stake shrinks to $\sigma$ from $\hat{\sigma}$, the economy reverts back to a frictionless permanent state. We will study equilibria in which the monetary

[9]It is easy to see that the introduction of a strictly convex labor disutility would for example imply that binding financial constraints depress the real rate.

authority utilizes the interest-rate rule:

$$R_t \begin{cases} = \hat{r}\left(\frac{P_t}{P_{t-1}}\right)^{1+\phi} & \text{during the financial crisis,} \\ = r_t\left(\frac{P_t}{P_{t-1}}\right)^{1+\phi} & \text{afterwards,} \end{cases} \tag{11}$$

with

$$\hat{r} = \frac{(1-\hat{\sigma})f(I^*)}{I^*}, \tag{12}$$

where

$$f'(I^*) = r^*. \tag{13}$$

In words, $\hat{r}$ is the value of the (real) interest rate such that a constrained entrepreneur would invest the quantity that an unconstrained entrepreneur reaches when the prevailing rate is $r^*$.

We now study the equilibria of this economy. We focus again on equilibria with bounded inflation.

**Proposition 2.** ***(Non-bubbly equilibrium)*** *There exists an equilibrium in which the price level is $P_{-1}$ and savers invest in entrepreneurs' projects a total amount $I^*$ during and after the crisis. Firms' output is equal to $I^* - e + u'^{-1}(\hat{r})$ during the crisis, and snaps back to the smaller value of the perfect-foresight model $I^* - e + u'^{-1}(r^*)$ afterwards. This is the only equilibrium if $\hat{r} > p$.*

**Proof.** See the appendix. ■

The interest rule (11) implies, as shown in the proof of Proposition 2, that the price path must be constant. This also entails that the central bank controls the real interest rate during the crisis, setting it at $\hat{r}$. Uncertainty resolves at the random date at which the financial crisis ends. The economy reverts to the frictionless perfect-foresight equilibrium with a real rate $r^*$ from then on.

The central bank can therefore use the interest rate to mitigate the financial friction and maintain entrepreneurs' "normal" borrowing capacity $I^*$ during the crisis from (12). This active monetary policy not only lifts entrepreneurs' demand for funds but also boosts output as $r^*$ is replaced with $\hat{r} < r^*$ in savers' Euler equation (9 ). It also raises the wage from (8).[10]

**Can there be bubbly equilibria as well?** To address this question, notice first that $r^* > 1$ implies that any bubble that might have risen during monetary easing must have burst no later than upon the end of it. The growth rate of the population is one, and so the expected return on a bubble

[10]Note that if $\hat{r}$ is strictly smaller than the value of the real rate under perfect competition $\gamma/(\alpha\beta)$, then the wage is larger than $\alpha$, and so firms must be subsidized in order to accept to pay such wages. The proof of Proposition 2 details this point.

is at most $p$ during monetary easing. This implies that there is no room for (stochastic) bubbles during the crisis if $\hat{r} > p$, in which case the non-bubbly equilibrium is unique. Conversely, if $\hat{r} \leq p$, monetary easing can spur bubbles. We construct a bubbly equilibrium under the following assumption:

**Assumption 4.** ***(Only financiers trade non-pledgeable bubbles)*** *Households cannot trade bubbles—for example because they are infinitely risk-avers when old. Financiers cannot borrow from the central bank against bubbles.*

The first part of this assumption captures that financiers, as the most sophisticated savers in this economy, are better at handling riskier assets than households.[11] The other component of the assumption is that the central bank does not accept bubbles as collateral.[12] This is realistic as central banks in all jurisdictions lend only against sufficiently high-quality assets as collateral—even though the set of admissible collateral has considerably expanded since 2008 and the Covid-19 crisis. In a real model, Hirano and Yanagawa (2016) rely on a similar assumption that bubbles are not pledgeable. Assumption 4 paves the way to bubbles that crowd out investments that yet would earn a higher total return. Notice that the non-bubbly equilibrium in Proposition 2 still holds under Assumption 4.

**Proposition 3.** ***(Bubbles against financial repression)*** *Suppose $\hat{r} < p$ and Assumption 4 holds. There exists an equilibrium in which a bubble $B$ is issued by old financiers at date 0, and then trades among financiers at a unit return during the crisis, and bursts when the crisis ends. Investment $I$ is smaller in this equilibrium than in the non-bubbly one. Furthermore, the monitoring constraint is binding and so households own a higher fraction of entrepreneurs' projects in the presence of the bubble than without it. The bubble depresses their output relative to the non-bubbly case. Finally, $(B, I, Y)$ are characterized by*

$$e = B + \frac{\mu f(I)}{p}, \tag{14}$$

$$\left(\frac{1-\sigma-\mu}{\hat{r}} + \frac{\mu}{p}\right) f(I) = I. \tag{15}$$

$$u'(Y - I + e - B) = \hat{r}. \tag{16}$$

**Proof.** See the appendix. ■

[11]The suggested microfoundation of infinitely risk-averse old households works because stochastic bubbles would be the only source of uncertainty that they face if investing in them. The rest of the analysis is thus not affected by this departure from the risk-neutrality assumed in (4).

[12]Section III relaxes this assumption.

Many other bubbly equilibria are of course sustainable when $\hat{r} < p$. Proposition 3 focuses on one of them that has a simple structure.[13] The bubble $B$ earns a unit return after its initial issuance by an old financier until it bursts at the end of the crisis. Thus, it keeps a constant size before bursting and earns an expected return $p > \hat{r}$.

Equations $\{$(14);(15);(16)$\}$ that pin down $(B, I, Y)$ are straightforward: (14) states that financiers split their savings into the bubble and their (minimum) stakes in entrepreneurs' projects; (15) reflects that investment size is driven by the binding incentive and monitoring constraints; and (16) is households' Euler equation.

Since financiers must have a minimum stake in entrepreneurs' projects, the superior expected return $p > \hat{r}$ on bubbles raises their hurdle rate and creates a higher cost of capital for entrepreneurs that negatively affects investment $I$ relative to the non-bubbly equilibrium in Proposition 2. Recall our assumption that in this non-bubbly equilibrium, the monitoring constraint is never binding: Financiers' stake strictly exceeds the minimum $\mu$ at all dates. By contrast, in the bubbly equilibrium in Proposition 3, entrepreneurs optimally make sure that financiers' stake binds because they no longer are indifferent between households' funds that command a return $\hat{r}$ and that of financiers commanding $p > \hat{r}$ in expectation. Leverage is thus higher in the presence of the bubble in the sense that the least sophisticated savers own a higher stake in entrepreneurs' projects than in the non-bubbly equilibrium.

The assumption that financiers cannot borrow from the central bank against the bubble is key in allowing for a superior return on bubbles as an equilibrium outcome. Otherwise, financiers would enter into "carry trades" on bubbles until the differential in expected returns vanishes.

Comparing the Euler equation (16) to its counterpart in the non-bubbly equilibrium:

$$u'(Y - I^* + e) = \hat{r}, \tag{17}$$

it is a priori unclear whether the bubble depresses or boosts output. On one hand, total investment is smaller in the bubbly equilibrium ($I < I^*$), which depresses output. On the other hand, the reduction $B$ in financiers's investment raises that of households and this boosts output. The proof of Proposition 3 shows that the dominant effect is a reduction in output because overall, the bubble reduces the pledgeability of entrepreneurs' projects towards households.

In sum, Proposition 3 delivers the goal stated in the introduction: exhibiting in the new Keynesian framework an example of a "bad bubble" that crowds out investments with superior returns as a result of savers' search for yield.

[13] Bubbles could have other sizes, earn other expected returns between $\hat{r}$ and $p$, could burst and rise stochastically at any date before the end of the crisis. Equations $\{$(14);(15)$\}$ at each date impose the restrictions on the joint dynamics of bubbles' creation and destruction, their expected returns, and entrepreneurs' investment. Given our goal in this paper to provide a simple example of a bad bubble, we do not fully characterize these dynamics and stick to the simplest bubble described in Proposition 3.

# III. Extensions and concluding remarks

## Pledgeable bubbles

The expected return on bubbles against financial repression can strictly exceed the policy rate only if financiers cannot borrow against them, as under Assumption 4. This assumption that monetary policy cannot reach all the corners of financial markets is realistic. Take the simple example of a non-financial corporation. Even if it can lever up and take on bank loans that (imperfectly) reflect policy rates, there is always an equity tranche on its future free cash flows that cannot be refinanced this way.

It is still instructive to discuss the case in which financiers can fully pledge their bubbles to the central bank. Absence of arbitrage entails that financiers earn the expected return $\hat{r}$ on all their investments. Entrepreneurs therefore face a cost of capital $\hat{r}$ during the crisis and invest at the central bank's target $I^*$. If financiers invest $e$ in entrepreneurs' projects and decide to borrow against a bubble $B$, the central bank must find resources to fund its loan. Suppose it taxes households a lump-sum amount $B$ to do so.[14] Then such carry trades on risky bubbles by financiers boost output by an amount $B$ relative to the non-bubbly equilibrium. To see this, notice that households' Euler equation:

$$u'(Y - B - I^* + e) = \beta\hat{r} \tag{18}$$

holds for a given $B$ if $Y$ raises by $B$ relative to the case in which $B = 0$.[15]

## Who issues the bubble?

Given our goal to construct bubbles that are as detrimental to investment as possible, we considered only the worst case in Proposition 3 in which the bubble is initially issued by an old financier who just consumes the proceeds. But the bubble could of course be initially issued by young entrepreneurs, who could use the proceeds to relax their incentive constraints, and this would boost initial investment. The subsequent rollover of the bubble among financiers would still crowd out high-return investments afterwards though.

Another possibility is that the initial proceeds from the bubble accrue to the central bank, who can then similarly transfer them to young entrepreneurs. This is akin to the central bank issuing reserves to finance "helicopter money" towards the private sector.

[14] Since the return on bubbles during the crisis is $\hat{r}/p < 1$, old financiers must issue $(1 - \hat{r}/p)B$ at each date for the bubble size to be constant.

[15] That there is no upper bound on feasible carry trades $B$ stems of course from the quasi-linear model with linear production.

This discussion about the initial benefiter from the bubble can be cast in the more concrete context of a stock market bubble following low rates. If the overall market bubble is mainly attached to the shares of firms with investment opportunities but financial constraints, then this initially helps them invest by boosting inside shareholders' wealth, as when young entrepreneurs issue the bubble in our model. If the overall bubble is non-directed, and spread over all shares, lots of the initial proceeds will not be efficiently invested this way. In this case it may be preferable that the central bank issues the bubble and direct the funds properly. This is however true only under the strong assumption that the central bank is better than the market at directing the proceeds towards constrained agents with investment opportunities. In sum, low rates create bubbles that are all the less desirable because they are not directed towards constrained firms with the best investment opportunities. If the public sector is better than the private one at directing these bubbles, then it should seek to evict private bubbles with large reserve issuances.

It is interesting to relate these results to the insights in Asriyan et al. (2020). In their paper, bubbles are efficient in the sense that they crowd out the right kind of investment and relax borrowing constraints, and so the public sector must let private bubbles grow and supply public bubbles in the form of money when the private ones burst. In our setup in which private bubbles crowd out the wrong kind of investment, the public sector may conversely seek to evict them with reserves.

## Policy implications

Our first policy implication is one that we share with the literature pointing at the adverse consequences from seeking to overcome financial frictions with low policy rates. Fiscal policies that directly subsidize constrained agents with valuable investment opportunities are preferable to interest policies when implementable. Such policies are straightforward in our environment as they merely consist in subsidizing entrepreneurs. The case against interest-rate policies is however weaker if one takes more realistically into account that i) fiscal measures may be politically difficult and too slow to implement in the face of a crisis; ii) they require that the fiscal authority have a lot of information in practice. In further defense of temporary interest-rate suppressions, notice that they are not conducive to bad bubbles in this setup if they are sufficiently punctual ($p$ sufficiently small other things being equal).

A second policy implication follows from the above discussion on the initial issuer of the bubble. If the central bank is not confident that the bubbles that it blows with its low rates are well directed, then it may try to evict private bubbles with bubbles on its liabilities. This means that the announcement of a reduction in interest rates must come with a large issuance of reserves.

Finally, and perhaps most novel, the above discussion about the pledgeability of bubbles speaks to central-banks' decisions to expand the list of collateral or/and assets that they are willing to purchase in times of crisis. Our analysis suggests that the carry-trade opportunities that result from pledgeability are useful as they reduce the possibility that some corners of the financial market stay immune to low rates and entertain bad bubbles that crowd out investment. The downside from such collateral expansion in terms of central-bank solvency and thus price stability (Hall and Reis, 2015, e.g.) is absent from our framework, however, in which the commitment to interest rules pins down stable price paths.

## Concluding remarks

This paper is an attempt at illustrating the widespread narrative that low policy rates may blow asset bubbles as monetary phenomena that crowd out investments with superior returns. This should be viewed foremost as a theoretical exercise, and a lot of additional work remains in order to determine whether such "bad" bubbles may be relevant to analyze the recent past, in which the natural rate was presumably low anyway. Yet, equilibria with bad bubbles have a number of interesting properties. First, monetary easing creates persistently high returns on risky assets, instead of a one-shot repricing when announced, in line with the fact that the S&P500 for example has had one of its very best decades in 140 years between 2008 and 2019. Events such as the 2013 taper tantrum suggest that low policy rates have been instrumental in sustaining such dynamics. Second, returns on existing real capital have also been much higher than interest rates and yet business investment has remained below historical trends over the period (Gomme et al., 2015; Furman, 2015), in line with crowding out. Finally, corporate leverage has reached historical highs in the past decade, and this resonates with our finding that entrepreneurs maximize the stake of households in their projects during the bubbly episode.

A route for future research consists in analyzing the ex-ante impact, during non-bubbly normal times, of the anticipation that monetary policy will be conducive to bubbles during future crises. Guerron-Quintana et al. (2020) show, in a real model, that the mere anticipation of future bubbles crowds out investment and is therefore detrimental to growth in such non-bubbly normal times.

# References


ALLEN, F., G. BARLEVY, AND D. GALE (2018): "A Theoretical Model of Leaning Against the Wind," Working paper No. 2017-16, Federal Reserve Bank of Chicago.

ASRIYAN, V., L. FORNARO, A. MARTÍN, AND J. VENTURA (2020): "Monetary Policy for a Bubbly World," Review of Economic Studies, Forthcoming.

BENMELECH, E. AND N. K. BERGMAN (2012): "Credit Traps," American Economic Review, 102, 3004–32.

CABALLERO, R. J. AND A. SIMSEK (2020): "A Risk-Centric Model of Demand Recessions and Speculation*," The Quarterly Journal of Economics, 135, 1493–1566.

CASTILLO-MARTINEZ, L. AND R. REIS (2019): "How do Central Banks Control Inflation? A Guide For the Perplexed," working paper.

DIAMOND, D. W. AND R. G. RAJAN (2012): "Illiquid Banks, Financial Stability, and Interest Rate Policy," Journal of Political Economy, 120, 552–591.

DIAMOND, P. A. (1965): "National Debt in a Neoclassical Growth Model," American Economic Review, 55, 1126–1150.

FARHI, E. AND J. TIROLE (2012a): "Bubbly Liquidity," Review of Economic Studies, 79, 678–706.

——— (2012b): "Collective Moral Hazard, Maturity Mismatch, and Systemic Bailouts," American Economic Review, 102, 60–93.

FURMAN, J. (2015): "Business investment in the united states: Facts, explanations, puzzles, and policies," Technical report, Council of Economic Advisers.

GALI, J. (2014): "Monetary Policy and Rational Asset Price Bubbles," American Economic Review, 104, 721–52.

GOMME, P., B. RAVIKUMAR, AND P. RUPERT (2015): "Secular Stagnation and Returns on Capital," Economic Synopses.

GUERRON-QUINTANA, P. A., T. HIRANO, AND R. JINNAI (2020): "Recurrent Bubbles and Growth," working paper.

HALL, R. E. AND R. REIS (2015): "Maintaining Central-Bank Financial Stability under New-Style Central Banking," CEPR Discussion Papers 10741, C.E.P.R. Discussion Papers.

HE, Z. AND A. KRISHNAMURTHY (2012): "A Model of Capital and Crises," The Review of Economic Studies, 79, 735–777.

——— (2013): "Intermediary Asset Pricing," American Economic Review, 103, 732–70.

HIRANO, T. AND N. YANAGAWA (2016): "Asset Bubbles, Endogenous Growth, and Financial Frictions," The Review of Economic Studies, 84, 406–443.

HOLMSTROM, B. AND J. TIROLE (1997): “Financial Intermediation, Loanable Funds, and the Real Sector,” The Quarterly Journal of Economics, 112, 663–691.

MARTIN, A. AND J. VENTURA (2012): “Economic Growth with Bubbles,” American Economic Review, 102, 3033–3058.

MEH, C. A. AND K. MORAN (2010): “The role of bank capital in the propagation of shocks,” Journal of Economic Dynamics and Control, 34, 555–576.

MIAO, J. AND P. WANG (2018): “Asset Bubbles and Credit Constraints,” American Economic Review, 108, 2590–2628.

TIROLE, J. (1985): “Asset Bubbles and Overlapping Generations,” Econometrica, 53, 1499–1528.

WOODFORD, M. (1990): “Public Debt as Private Liquidity,” American Economic Review, 80, 382–388.

# Appendix

## A.1. Microfoundation for $\mu$ and $\sigma$

Suppose that an entrepreneur privately selects one among three technologies in which to invest resources $I$:

1. A technology that delivers $f(I)$ consumption units;
2. A technology that delivers no consumption units and a private benefit $\sigma f(I)$
3. A technology that delivers no consumption units and a private benefit $f(I)$. A fraction $\mu$ of this latter private benefit can be transferred to a financier though.

Financiers can prevent entrepreneurs from choosing technology 3.

We suppose throughout that $e$ is sufficiently small that entrepreneurs find it optimal to raise funds from households. They must therefore be incentivized to choose technology 1. A financier can credibly block an entrepreneur from using technology 3 if she has incentives to do so, which is the case if she has a stake at least $\mu$ in the project. In this case the entrepreneur chooses 1 with a stake at least $\sigma$ in the project.

## A.2. Proof of Proposition 1

As is standard, combining the Fisher equation and the interest rule yields for all $t \geq 0$

$$\frac{P_t}{P_{t-1}} = \left(\frac{P_{t+1}}{P_t}\right)^{\frac{1}{1+\phi}} \tag{A.1}$$

and so the only non-explosive price path corresponds to the target unit inflation, implying $P_t = P_{-1}$ for all $t \geq 0$.

Furthermore, given perfect foresight, the information set is constant, and so the fact that prices are set in advance is immaterial: The economy is identical to a flexible-price one.

When posting price $P_i$, firm $i \in [0, 1]$ faces demand $Y_i = Y(P/P_i)^{\epsilon}$, where $Y$ is the output of the final good. Taking the wage $W$ as given, profit maximization then reads

$$\max_{P_i} \Pi_i \equiv P_i Y_i - \frac{WY_i}{\alpha}, \tag{A.2}$$

and $P_i = P$ in equilibrium yields the real wage

$$w = \frac{\alpha(\epsilon - 1)}{\epsilon}. \tag{A.3}$$

Financiers invest their entire endowment $e$ with entrepreneurs. Saver $i \in [0,1]$ solves

$$\max_{C_Y, C_O, L} u(C_Y) + \beta C_O - \gamma L \tag{A.4}$$

s.t.

$$C_Y + \frac{C_O}{r} = wL + \frac{\Pi_i}{P}, \tag{A.5}$$

$$C_Y, C_O, L \geq 0, \tag{A.6}$$

and in equilibrium

$$wL + \frac{\Pi_i}{P} = Y = \alpha L, C_Y = Y - I + e. \tag{A.7}$$

where $r$ is the real interest rate and $I$ denotes borrowers' investment. Optimal labor supply yields

$$wu'(Y - I + e) = \gamma, \tag{A.8}$$

The Euler equation is

$$u'(Y - I + e) \geq \beta r, \tag{A.9}$$

binding if $C_O > 0 \iff e < I$, which is the case for $e$ sufficiently small other things being equal. This yields the real rate $r = r^*$ . Finally, borrowers' optimality yields $I$ from

$$\begin{cases} (1 - \sigma - \mu) f(I) = r^*(I - e) \text{ if both the incentive and monitoring constraints bind,} \\ (1 - \sigma) f(I) = r^* I \text{ if only the incentive constraint binds,} \\ (1 - \mu) f'(I) = r^* \text{ if only the monitoring constraint binds,} \\ f'(I) = r^* \text{ otherwise.} \end{cases} \tag{A.10}$$

This uniquely defines the non-bubbly perfect-foresight equilibrium and there is no bubbly one since $r^* > 1$.

## A.3. Proof of Proposition 2

We first show that the price level is constant. Suppose that the crisis ends at date $T$. The economy is then in the equilibrium in Proposition 1 without binding constraints. The interest-rate rule implies again that $P_t = P_{T-1}$ for all $t \geq T$ . Since there is always a strictly positive probability that the crisis will end at date $t$, the only price levels that are consistent with the initial $P_{-1}$ and this relation after the crisis is constant, equal to $P_{-1}$.

During the crisis, the real rate $\hat{r}$ implies that investment is $I^*$ from (10). The binding Euler equation (A.9) yields $Y$, and (A.8) yields $w$. Notice that firms are willing to produce $Y$ only if $w < \alpha$, which is not the case as soon as

$$\hat{r} = \frac{\gamma}{\beta w} < \frac{\gamma}{\alpha\beta}, \tag{A.11}$$

the real rate when $\epsilon = +\infty$. If parameters lead to such a situation, the public sector must subsidize firms' prices. This subsidy can be financed with a lump-sum tax on households.

# A.4. Proof of Proposition 3

All that is left from the body of the paper is the proof that the bubble depresses the output. Comparing (16 ) and (17) yields that this is the case if and only if

$$I^* - e \geq I - e + B, \tag{A.12}$$

equivalent to

$$\frac{(1-\sigma)f(I^*)}{\hat{r}} \geq \frac{(1-\sigma-\mu)f(I)}{\hat{r}}, \tag{A.13}$$

which always holds.