The Central Bank, the Treasury, or the Market: Which One Determines the Price Level?*

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Abstract

This paper studies a model in which the price level is the outcome of dynamic strategic interactions between a fiscal authority, a monetary authority, and investors in government bonds and reserves. The "unpleasant monetarist arithmetic" whereby aggressive fiscal expansion forces the monetary authority to chicken out and inflate away public liabilities may be contained by market forces: Monetary dominance prevails if such fiscal expansion is met with a higher real interest rate on public liabilities, due for example to the crowding out of private investment opportunities. We derive policy implications regarding central-bank balance sheet management and the need for fiscal requirements.

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1 Introduction

Public sectors in most major economies have issued since 2008 an amount of liabilities, both government debt and central-bank reserves, that is unprecedented in peacetime. Their resulting fiscal positions have led a number of observers to worry about the ability of central banks to fulfill the price-stability part of their mandates going forward.

The theoretical underpinning of this worry can be traced back to Sargent and Wallace's "unpleasant monetarist arithmetic" (Sargent and Wallace, 1981). This seminal paper shows that if a fiscal authority embarks on a path of aggressive debt issuance and deficits, the monetary authority has no option but generating sufficient seigniorage income despite the inflationary consequences if it cares about sovereign solvency. This seminal paper has initiated a large body of research studying the respective contributions of fiscal and monetary policies to the determination of the price level.

Wallace has famously described fiscal and monetary interactions as a "game of chicken" between the branches of government respectively in charge of fiscal and monetary policies. This paper takes this view seriously and develops a full-fledged dynamic strategic analysis of the determination of the price level. We write down a model that features a fiscal authority, a monetary one, and a private sector that interact strategically. The monetary authority seeks to control the price level. It issues reserves that are the unit of account of the economy: The price of consumption units in terms of reserves is the price level. The monetary authority decides on the nominal interest rate on reserves, on the investment of the proceeds from issuing reserves, and on possible transfers ("dividends") to the fiscal authority. The fiscal authority seeks to spend optimally. It issues nominal bonds and uses the proceeds to spend or/and to repay all or part of maturing bonds. Walrasian private investors form optimal portfolio of reserves, government bonds, and private investments.

We solve for the (subgame-perfect) Nash equilibria resulting from their interactions with a focus on the resulting price level. We deem "monetary dominance" the situation in which the equilibrium price level corresponds to the target of the monetary authority. "Fiscal dominance" is the alternative in which the price level jumps above this target, and reaches instead the lowest level that is consistent with the solvency of the public sector.

Two departures from Sargent and Wallace (1981) play a central role in our main insights. First, an implicit assumption in their paper is that the fiscal authority "moves first" in the sense that it can commit to a path of debt issuance and deficits for the entire future. As a second mover, the monetary authority then has to accommodate this path. By contrast, all agents repeatedly interact without commitment in our model, and so who "moves first" and imposes its objectives in equilibrium is endogenously driven by the primitives of the economy. Second, the government faces an infinitely elastic demand for bonds in Sargent and Wallace (1981). By contrast, bond and reserve issuances push up the real interest rate in our model.[1]

The reason these two features of the model play an important role is as follows. The fact that the fiscal authority cannot commit to future deficits implies that if it wants to force the monetary authority to "chicken out" and inflate away public liabilities as in Sargent and Wallace (1981), it must credibly eliminate any future fiscal capacity by borrowing now against any future resources and spending the proceeds right away. This may require a large issuance of government bonds. Such a large issuance in turn pushes the (real) interest rate at a higher level than the one that would prevail if the fiscal authority was not seeking to impose fiscal dominance this way. If the cost from borrowing such large amounts at such a high rate offsets the benefits from forcing the monetary authority to inflate away legacy liabilities, then the fiscal authority does not enter into this "Sargent-Wallace" behavior, and there is monetary dominance. Remarkably, in this case, the central bank, despite having neither commitment power nor fiscal support, can fulfill its price-level mandate. The only commitment that is required from the government is that it lets the central bank manage its balance sheet independently and, of course, that it refrains from renegotiating its mandate.[2] Otherwise there is fiscal dominance, and the price level is dictated by sovereign solvency, echoing the fiscal theory of the price level. In sum, one may describe our contribution as an answer to the question that Sargent and Wallace (1981) raise in conclusion of their unpleasant arithmetic: *"The question is, Which authority moves first, the monetary authority or the fiscal authority? In other words, Who imposes discipline on whom?"* We show that the monetary authority imposes its views if and only if sufficiently strong market forces imply that any fiscal victory in the "game of chicken" must be a Pyrrhic one via an excessively high real interest rate.

Since monetary dominance arises when the gain from inflating away legacy liabilities is small and the cost from spending future tax capacity right away is large, it is more

[1]Specifically, they do so by crowding out private investment. Yet any other reason for a downward-sloping demand for public securities would have the same implications.

[2]Presumably, reneging on central-bank independence is politically more costly and institutionally more complex than merely embarking on aggressive fiscal expansion.

likely to prevail under the following conditions: small legacy liabilities, profitable private investment opportunities that entail a large impact of crowding out on the interest-rate level, a large future tax capacity, and a "patient" fiscal authority. Fiscal dominance prevails otherwise.

These forces generate interesting joint dynamics for public finances, the real interest-rate level, and the price level. The regime may switch from monetary to fiscal dominance over time as the "net wealth" of the government, which is in turn driven by the endogenous interest rate, decreases. The equilibrium may in particular be such that interest rates are low and price levels on target despite large public debt and deficits for a long period of time, at the end of which inflation picks up and fiscal consolidation arises.

Finally, we study a version of the model in which dynamic inefficiency enables the public sector to issue unbacked reserves and bonds—pure bubbles. Of course, there are in this case multiple equilibria. We construct in particular equilibria in which the private sector can enforce any price level by credibly threatening to prick the bubbles on public liabilities if the public sector deviates from this level. This interference of market discipline with fiscal and monetary interactions, leading to a situation of "market dominance," is novel to our knowledge.

Our model has several policy implications, on the normative side to start with. First, fiscal requirements in the form of a cap for debt can substitute market forces to discipline the fiscal authority—however, these fiscal requirements may be time-inconsistent, especially in high-debt environments. Second, to ensure monetary dominance, our model emphasizes the need for monetary authorities to hoard sufficient future resources that do not depend on the government's solvency, and so the composition of the asset side of their balance sheet matters.

On the positive side, our paper emphasizes that the net public liabilities in the hands of the private sector are the key variable to keep track of the risk of fiscal dominance. Second, our paper emphasizes that the game of chicken has an important timing component whereby public debt increases before inflation picks up.

Related literature. Our paper belongs to the very rich literature on optimal fiscal and monetary policies following Calvo (1978) and Lucas and Stokey (1983). As envisioned in this literature, nominal public liabilities lead to a time-inconsistency problem for public authorities. Furthermore, this literature has also discussed the importance for this time-inconsistency problem of the public sector's net nominal liabilities, i.e., nominal debt and money in the hands of the private sector (see Alvarez et al., 2004; Persson et al., 2006, among others). In our framework, delegation of monetary tools to the monetary authority helps solve the time-inconsistency of the government, but imperfect delegation due to limited commitment creates a game between fiscal and monetary authorities.

From this perspective, we are connected to the literature on the interactions between monetary and fiscal policies pioneered by Sargent and Wallace (1981) (see Leeper, 1991; Sims, 1994; Woodford, 1994, 1995; Cochrane, 2001, 2005; McCallum, 2001; Buiter, 2002; Niepelt, 2004; Jacobson et al., 2019, among others). As in Sargent and Wallace (1981), the monetary authority can adjust seignorage revenue to help the fiscal authority satisfy its budget constraint. The simple economy in which we cast our game of chicken relates in particular to one of the models in Bassetto and Sargent (2020), in which public liabilities also serve as liquidity vehicles. Our paper is also closely connected to the papers that identify fiscal requirements such that the central bank can attain its price stability objective, including fiscal rules (e.g. Woodford, 2001) or a ring-fenced balance sheet (e.g Sims, 2003; Bassetto and Messer, 2013; Hall and Reis, 2015; Benigno, forthcoming). Closer to our paper, Martin (2015) finds as we do that fiscal irresponsibility leads to long-term inflation. Finally, Coibion et al. (2021) provide causal evidence that private agents do anticipate inflationary effects of fiscal policy: Their evidence that households associate future debt levels with inflation is consistent with our model's result that future net public liability is a key determinant of central bank's future incentives to inflate. In line with this literature, our paper aims to precisely describe the markets, the instruments and the budget constraints of the two authorities. Our contribution is to explicitly model the strategic interactions between fiscal and monetary authorities in such an environment.

That fiscal and monetary authorities may have ex-post conflicting objectives is a natural assumption. This has been in fact the main rationale behind setting up independent central banks. This is also motivated by the large set of evidence that authorities do not necessarily cooperate and, instead, try to impose their views on each other (see Bianchi et al., 2019, among others), even though coordination dominates (see Bianchi et al., 2020, for a recent contribution). In this respect, this makes our paper closer to an older literature (Alesina, 1987; Alesina and Tabellini, 1987; Tabellini, 1986, e.g.) that investigates the equilibria of games between multiple branches of government. More recent contributions include Dixit and Lambertini (2003) or the literature that explores disciplining mechanisms for the public sector in models following Barro and Gordon (1983a,b), such as Halac and Yared (2020).

With respect to this literature, our contribution is to provide an explicit set of instruments to both the fiscal and the monetary authorities as well as a game-theoretic foundation to fiscal and monetary interactions. Our approach of the resulting macroeconomic game follows Chari and Kehoe (1990), Stokey (1991) and Ljungqvist and Sargent (2018) but extended to multiple large agents and markets. In particular, our approach to model markets follows Bassetto (2002) as, in our setting, price levels as well as debt prices are market equilibrium objects.

Finally, our paper relates to the literature building on the idea that public debt satisfies private liquidity demand. This literature goes back to Diamond (1965) and has been widely studied since (see Woodford, 1990; Aiyagari and McGrattan, 1998; Holmström and Tirole, 1998, among others). Krishnamurthy and Vissing-Jorgensen (2012) show in the data that public debt shares many of the properties of money. More recent contributions on optimal public liquidity supply include Angeletos et al. (2020), Azzimonti and Yared (2019) or Gorton and Ordonez (2021). Our paper extends some of the insights of this literature to a context where multiple authorities can issue liquidity vehicles and behave strategically. In addition, we investigate both cases where public liabilities are backed by real resources and where they are unbacked and stem from a bubble. Related to this literature, some recent contributions investigate the implication of bubbles on monetary policy (see Galí, 2014; Asriyan et al., 2019, among others) and on fiscal/monetary interactions (Bassetto and Cui, 2018; Brunnermeier et al., 2020). We show that when public liquidity supply is a self-fulfilling phenomenon, monetary or fiscal dominance is essentially driven by the private sector's expectations—a situation that we deem "market dominance".

2 Model

Our model features a fiscal authority and a monetary one that interact strategically. They also interact with the private sector in the markets for their respective liabilities.

The monetary authority issues reserves that are the unit of account of the economy and seeks to control the price level. The fiscal authority seeks to consume optimally and issues nominal bonds.

2.1 Setup

Time is discrete. There is a single consumption good. The economy is populated by a private sector and by a public one.

Private sector. At each date, a unit mass of agents, deemed "savers", are born. They live for two dates and value consumption only when old, at which time they are risk-neutral. They are each endowed with one unit of the consumption good when young. A storage technology is available to savers at each date. Each saver can transform $x > 0$ consumption units into $f(x)$ units at the next date. We suppose that $r(.) \equiv f'(.)$ exists and is a decreasing, strictly convex bijection mapping $(0, 1]$ into $[r(1), +\infty)$.[3]

Public sector. The public sector features a fiscal authority F and a monetary authority M.

Monetary authority. The monetary authority issues reserves and sets the (gross) nominal interest rate R_t on them. Reserves are claims of infinite maturity. A unit of reserves at date t is a claim to R_t units of reserves at date $t + 1$. Reserves are the unit of account of the economy, and can be traded for the consumption good in the market for reserves. We denote by P_t the price level—the date-t price of the consumption good in terms of reserves in the market for reserves, by $X_t \geq R_{t-1}X_{t-1}$ the quantity of outstanding reserves at the end of date t (resulting from cumulative past issuances between 0 and t), and by x_t the quantity of goods that savers bid for reserves in the date-t market for reserves.

M can also transfer resources to F ("pay a dividend"), and θ_t denotes the real date-t transfer from M to F.

[3]Here, we assume decreasing returns on storage at the individual level. Our framework and results readily extend to the alternative assumption of decreasing returns at the aggregate level, in which case each individual saver's return on storage is linear.

Fiscal authority. The fiscal authority issues one-period nominal bonds. A bond issued at date t is a claim to one unit of reserves at date $t+1$. Both savers and M can trade goods for bonds. Let B_t denote the number of bonds issued by F at date t, Q_t the price at which they are sold (in terms of reserves), and b_t and b_t^M the respective quantities of goods that savers and M respectively trade for bonds in the bond market.

F decides at each date t on the haircut or loss given default $l_t \in [0,1]$ that it applies to the bonds maturing at date t. A haircut l means that bondholders receive $(1-l)$ units of reserves per bond. F also consumes. Let g_t denote its date-t consumption. Consumption can also be interpreted as net spending—transfers towards the private sector or a subset of it— without affecting the results.

Summary of notations. We introduced the following variables:

Interest rate on reserves set by M	R_t
Outstanding reserves at the end of date t	X_t
Goods invested by savers in the market for reserves	x_t
Price level	P_t
Bonds issued by F	B_t
Goods invested by M in the bond market	b_t^M
Goods invested by savers in the bond market	b_t
Bond price	Q_t
(Real) transfer from M to F	θ_t
Haircut on maturing bonds by F	l_t
Consumption of F	g_t

Let $\mathcal{E}_t = (R_t, X_t, x_t, P_t, B_t, b_t^M, b_t, Q_t, \theta_t, l_t, g_t)$ denote the vector of all the variables that describe the economy at date t. Appendix A states the conditions for a sequence $(\mathcal{E}_t)_{t\in\mathbb{N}}$ to form a competitive equilibrium. The remainder of the paper takes another route and studies full-fledged strategic interactions between the agents. The equilibrium paths $(\mathcal{E}_t)_{t\in\mathbb{N}}$ resulting from these interactions will all form a competitive equilibrium, though.

The rest of this section outlines the game in a standard fashion. We first define the objectives of the agents. We then present the extensive form of the game. We finally state our equilibrium concept, which is that in Ljungqvist and Sargent (2018). In order to encompass versions of the model with both finite and infinite horizons, we introduce a terminal date $T \in \mathbb{N} \cup \{+\infty\}$.

2.2 Objectives

Young savers' objective. Young savers born at $t < T$ seek to maximize their expected consumption at $t+1$.[4]

Objectives of F and M. For all $t < T$ and at T if $T \in \mathbb{N}$, the respective date-t objectives of F and M are:

$$U_t^F = \sum_{s=t}^{T} \beta^{s-t}(g_s - \alpha_F \Delta_s), \tag{1}$$

$$U_t^M = -\sum_{s=t}^{T} \beta^{s-t}(\mid P_s - P_s^M \mid + \alpha_M \Delta_s), \tag{2}$$

where $\Delta_s = \mathbb{1}_{\{l_s>0\}}$, $\beta \in (0,1)$, $\alpha_F, \alpha_M > 0$, and $P_s^M > 0$. In words, the variable Δ_t is equal to 1 in case of an outright default on a government bond due at date t, and to 0 otherwise.

In sum, each authority $X \in \{F; M\}$ incurs a cost α_X in case of sovereign default.[5] The fiscal authority also values consumption (but does not care about the price level), whereas the monetary authority also finds it costly to deviate from a given target P_t^M for the date-t price level.[6]

We focus for brevity on the case in which α_F is arbitrarily large. In other words, F is willing to do whatever it takes to avoid sovereign default. For expositional brevity as well, we assume the following lexicographic preferences for M. Holding (2) fixed, M prefers to maximize (1). These lexicographic preferences will play only a tie-breaking role that we will explain in due course.

[4]Notice that, in our setting, reserves do not provide additional liquidity services relative to bonds. This assumption involves no loss of generality and is made for tractability purposes: Such additional services would allow the central bank to extract more resources through seignorage, which would exacerbate the temptation for F to enter in the game of chicken.

[5]Costs from outright default are exogenous here. Section 4 discusses equilibria in which savers create endogenous default costs.

[6]Results would be similar with an inflation target. Section 3.3 explains why the creation of a monetary authority with such an objective is ex-ante desirable.

2.3 Extensive-form game

For a given date $0 \leq t < T$, consider a history $h_t = (R_s, X_s, x_s, B_s, b_s^M, b_s, l_s, g_s)_{s<t}$.[7] Old date-$t$ savers sell reserves, collect bond repayments and proceeds from storage, and consume. At date 0 in particular, old savers sell reserves $R_{-1}X_{-1} > 0$, and, for simplicity, we assume away any legacy bonds ($B_{-1} = 0$). The other agents—F, M, and young savers—interact as follows at date t. As is standard, the notation $a(b)$ below means that action a is conditional on the information set b. A strategy profile must then describe for each action a the mapping $a(.)$ of every possible information set into an action choice. We deem "action" of the private sector the aggregate quantity that it invests in reserve and bond markets, a natural abuse of language given our equilibrium concept below. Date-t is split into three consecutive stages: the reserve market, the bond market, and default and consumption decisions by F.

Stage 1: Market for reserves.

1. M selects $R_t(h_t) \geq 0$ and $X_t(h_t) \geq R_{t-1}X_{t-1}$, issuing new reserves $X_t(h_t) - R_{t-1}X_{t-1}$ on top of $R_{t-1}X_{t-1}$ sold by old savers.

2. Young savers invest an aggregate quantity $x_t(h_t, R_t, X_t) \in [0, 1]$ of consumption units in the market for reserves. The price level P_t is given by $P_t x_t = X_t$, with the convention that it is infinite if $x_t = 0$.

Stage 2: Bond market.

3. F issues $B_t(h_t, R_t, X_t, x_t) \geq 0$ bonds.

4. M invests $b_t^M(h_t, R_t, X_t, x_t, B_t) \in [0, (X_t - R_{t-1}X_{t-1})/P_t]$ consumption units in the bond market.

5. Young savers invest $b_t(h_t, R_t, X_t, x_t, B_t, b_t^M) \in [0, 1 - x_t]$ aggregate consumption units in the bond market. The bond price Q_t is given by $Q_t B_t = P_t(b_t + b_t^M)$, with the convention that it is infinite if $B_t = 0$.

[7] Notice that $(P_s)_{s<t}$ and $(Q_s)_{s<t}$ are not in h_t because, as shown below, they are derived from h_t out of market-clearing conditions. Nor is $(\theta_s)_{s<t}$ which is also given by h_t, and by the flow budget constraint of M.

Stage 3: Default and consumption.

6. F selects a haircut on maturing bonds $l_t(h_t, R_t, X_t, x_t, B_t, b_t^M, b_t) \in [0,1]$ and consumption $g_t(h_t, R_t, X_t, x_t, B_t, b_t^M, b_t) \geq 0$ such that

$$Q_t B_t + P_t \theta_t = P_t g_t + (1 - l_t) B_{t-1}, \tag{3}$$

where

$$\theta_t = \frac{X_t - R_{t-1} X_{t-1}}{P_t} - b_t^M + \frac{(1 - l_t) b_{t-1}^M P_{t-1}}{Q_{t-1} P_t}. \tag{4}$$

A date-t strategy profile $\sigma_t = (R_t, X_t, x_t, B_t, b_t^M, b_t, l_t, g_t)$ describes all the above date-t actions of each agent given all possible information sets.

If $T = +\infty$ then this generic date t fully describes the extensive form of the infinite-horizon game. Otherwise, there is also a terminal date T:

Terminal date T. If $T \in \mathbb{N}$, then no savers are born at T and this terminal date T features two stages. Let $\bar{x}, \bar{b} > 0$.

1. M receives an exogenous terminal demand for reserves $\bar{x}$ from unmodelled agents and issues $X_T(h_T) - R_{T-1} X_{T-1} \geq 0$. The price level P_T solves $P_T \bar{x} = X_T$.

2. F receives an exogenous fiscal income $\bar{b}$, and decides on $l_T(h_T, X_T) \in [0,1]$ and $g_T(h_T, X_T) \geq 0$ such that

$$g_T = \bar{b} + \theta_T - \frac{(1 - l_T) B_{T-1}}{P_T}, \quad \theta_T = \frac{X_T - R_{T-1} X_{T-1}}{P_T} + \frac{(1 - l_T) b_{T-1}^M P_{T-1}}{Q_{T-1} P_T}. \tag{5}$$

A strategy profile for the whole game is a sequence $\sigma = (\sigma_t)_{t \leq T}$ if $T \in \mathbb{N}$ and $\sigma = (\sigma_t)_{t \in \mathbb{N}}$ otherwise.

Remarks. Two remarks are in order. First, the assumption that F is first-mover in the bond market (formally B_t is in the information set of M when it decides on b_t^M) is only to fix ideas: The results are similar when M moves first instead in the bond market. Similarly, the order of b_t^M and b_t is immaterial.[8] Second, F makes haircut and consumption decisions understanding that the transfer θ_t that it receives from M is affected by the haircut. In other words, F must satisfy its flow budget constraint (3) when choosing l_t and g_t understanding that θ_t must satisfy that of M given by (4).

Relationship to the competitive equilibrium. Five relations define a standard competitive equilibrium in Appendix A: reserve and bond market clearing, the flow budget constraints of F and M, and the requirement that savers invest optimally. The flow-budget constraints are built in the action sets of F and M and so are satisfied for all feasible actions, on and off the equilibrium path, from (3) and (4). Similarly, reserve and bond market clear on and off the equilibrium path by construction of P_t and Q_t. The last condition, the optimal behavior of price-taking savers, is part of the equilibrium definition that follows.

2.4 Equilibrium concept

Definition 1. ***(Equilibrium)*** *An equilibrium is a strategy profile σ such that:*

1. *Each action by F and M is optimal given its information set and its beliefs that the future actions are taken according to the strategy profile.*

2. *Date-t young saver $i \in [0,1]$ optimally invests $x_t^i = x_t$ in the reserve market given (h_t, R_t, X_t), P_t, and the strategy profiles for all future actions, and optimally invests $b_t^i = b_t$ in the bond market given $(h_t, R_t, X_t, x_t, B_t, b_t^M)$, Q_t and the strategy profiles for all future actions.*

Our equilibrium concept is that of Ljungqvist and Sargent (2018), which adapts plain game-theoretic subgame perfection to the situation in which a "large" player interacts with Walrasian agents. We extend this concept to the case in which there are two such large players, a monetary and a fiscal authority. Very intuitively, F and M play against "the private sector," which responds to their supply of reserves and bonds with aggregate demands in reserve and bond markets. Reserve and bond prices then result from market clearing. In equilibrium, these "actions" of the private sector correspond to prices and

[8] All that matters is that M and savers do not move simultaneously in the bond market as this would generate multiple equilibria.

aggregate quantities that are consistent with optimal behavior by each individual saver given fiscal and monetary policies. Appendix C offers a formal version of this equilibrium definition that formally spells out the objective of each agent at each step.

Backed versus unbacked public liabilities. It is important to stress that in the finite-horizon version of the model, the exogenous demand for money $\bar{x}$ and fiscal revenue $\bar{b}$ will back reserves and bonds. The incompleteness inherent to overlapping generations plays no role in the rise of public liabilities, and we could dispense with it. Conversely, the infinite-horizon model assumes away such backing and public liabilities must be bubbles enabled by dynamic inefficiency. We could consider a third case in which the public sector has real revenue and the horizon is infinite, possibly creating room for a bubbly component in the price of backed public liabilities. We would however not gain any insight relative to the two polar cases studied here—finite horizon with backed liabilities and infinite horizon with unbacked liabilities.

The rest of the paper analyzes the game in three steps. We first solve for the finite-horizon game with two dates ($T = 1$). This enables us to introduce the central insights of the paper in the simplest environment. We then extend the two-dates analysis to all finite games. Finally, we tackle the infinite-horizon game.

3 Finite-horizon equilibria

Subgame perfection boils down to sequential rationality when $T \in \mathbb{N}$ and so we can solve finite games using backwards induction.

3.1 Two-dates game $T = 1$

Suppose there are two dates ($T = 1$). At the terminal date 1, M and F play as follows given history $h_1 = (R_{-1}, X_{-1}, R_0, X_0, x_0, B_0, b_0^M, b_0, l_0, g_0)$.

Lemma 1. ***(Terminal date 1)*** *If*

$$B_0 - b_0^M P_0/Q_0 + R_0 X_0 \le (\bar{x} + \bar{b}) \left(\max \left\{ P_1^M ; \frac{R_0 X_0}{\bar{x}} \right\} + \alpha_M \right), \tag{6}$$

then M sets the price level

$$P_1 = \max\left\{P_1^M; \frac{R_0X_0}{\bar{x}}; \frac{B_0 - \frac{b_0^M P_0}{Q_0} + R_0X_0}{\bar{x}+\bar{b}}\right\} \tag{7}$$

by setting $X_1 = \bar{x}P_1$. F fully repays maturing bonds: $l_1 = 0$, and consumes $g_1 = \bar{x} + \bar{b} - \left(B_0 - b_0^M P_0/Q_0 + R_0X_0\right)/P_1$.

Otherwise, M sets

$$P_1 = \max\left\{P_1^M; \frac{R_0X_0}{\bar{x}}\right\}, \tag{8}$$

F fully defaults on B_0: $l_1 = 1$, and consumes $g_1 = \bar{x} + \bar{b} - R_0X_0/P_1$.

Proof. See Appendix B.1. □

Expressions (7) and (8) show that the date-1 price level P_1 is always (weakly) above target P_1^M. There are two cases in which M might have to go strictly above its target P_1^M:

The case of "reserve overflow" when $P_1 = R_0X_0/\bar{x} > P_1^M$. First, the reserves sold by old savers R_0X_0 might be strictly larger than $\bar{x}P_1^M$, in which case the price level must be at least equal to $R_0X_0/\bar{x} > P_1^M$. We will see below that, given the perfect-foresight environment and in the absence of a zero lower bound on the interest rate, this never occurs along the equilibrium path.[9]

The case of "fiscal dominance" when $P_1 = (B_0 - b_0^M P_0/Q_0 + R_0X_0)/(\bar{x}+\bar{b}) > P_1^M$. Second, averting default may require a price level P_1 sufficiently large that the total resources of the public sector, $\bar{x}+\bar{b}$, suffice to repay the total public liabilities in the hands of the private sector, $(B_0 - b_0^M P_0/Q_0 + R_0X_0)/P_1$. This situation is one of fiscal dominance in which the price level strays away from the target of the monetary authority, and is dictated by the overall budget constraint of the public sector.

Lemma 1 also shows that if the cost for M to avert default exceeds α_M, then M prefers to set the price at $\max\{P_1^M; R_0X_0/\bar{x}\}$, and to let F default. The cost of averting default

[9] Except in the case in which M deliberately uses this to commit to a date-1 price level that it finds ex-post excessive (see Proposition 4 below).

exceeds α_M when (6) fails to hold. We will see below that such default never occurs in equilibrium, but that the solvency constraint (6) plays a central role in the strategy of F.

We are now equipped to describe date 0.[10] Consider first the third stage in which F makes consumption and default decisions. In the absence of maturing debt ($B_{-1} = 0$) there is no haircut decision. M simply transfers $x_0 - R_{-1}X_{-1}/P_0 - b_0^M$ to F who consumes it on top of the amount $b_0 + b_0^M$ collected in the bond market. F thus consumes $x_0 + b_0 - R_{-1}X_{-1}/P_0$, independent of b_0^M.

The next Lemma then describes the action of F at stage 2 of date 0, the bond market, after it has observed (R_0, X_0, x_0) from the date-0 reserve market. Notice that these latter values are arbitrary, possibly off the equilibrium path. From Lemma 1, if F issues B_0 bonds, the date-1 continuation will be either such that there is fiscal dominance or not. The following lemma describes the optimal issuance levels chosen by F conditionally on each of these two continuation outcomes. Which of these two conditionally optimal actions is unconditionally optimal and taken by F in equilibrium will then be stated in the subsequent Proposition 3 that describes the whole equilibrium path.

Lemma 2. ***(Debt issuance in the date-0 bond market)*** *Given* (R_0, X_0, x_0)*,* F *issues one of either debt level:*

- ***Price-level taking debt level:*** *F issues bonds so as to optimize its consumption pattern taking the date-1 price* $\max\{P_1^M; R_0X_0/\bar{x}\}$ *as given. In this case* $g_1 > 0$ *and M does not intervene in the bond market but pays a dividend to F at date 0 with its resources* $x_0 - R_{-1}X_{-1}/P_0$ *(if any). There is no default at date 1.*

- ***Sargent-Wallace debt level:*** *F issues a large amount in the bond market, frontloading consumption as much as possible* ($g_1 = 0$)*, so as to force a date-1 price level given by fiscal dominance. M buys back as many bonds as possible:* $b_0^M = x_0 - R_{-1}X_{-1}/P_0$*, but not the whole issuance. The date-1 price level is above target, equal to* $\max\{P_1^M; R_0X_0/\bar{x}\} + \alpha_M$*. There is no default at date 1.*

Proof. See Appendix B.1. ☐

When selecting the "price-level taking" debt level, F acknowledges that its bond issuance B_0 will not affect the date-1 price: It anticipates $P_1 = \underline{P}_1 \equiv \max\{P_1^M; R_0X_0/\bar{x}\}$.

[10] Appendix B.1 offers a full formal derivation of the equilibrium. The body of the paper describes its most important features.

In other words, F admits date-1 monetary dominance. It then seeks to optimally consume taking the price levels as given, and thus issues $B_0 = \underline{P}_1 r(1-\hat{b}(x_0)-x_0)\hat{b}(x_0)$, where

$$\hat{b}(x_0) \equiv \arg\max_{b}\{g_0 + \beta g_1\} \tag{9}$$

s.t.

$$g_0 = x_0 + b - \frac{R_{-1}X_{-1}}{P_0}, \tag{10}$$

$$g_1 = \bar{x} + \bar{b} - \frac{R_0 X_0}{\underline{P}_1} - r(1-x_0-b)b, \tag{11}$$

$$0 \leq b < 1 - x_0, 0 \leq g_1. \tag{12}$$

We denote $\hat{g}_1(x_0)$ the date-1 consumption of F resulting from this program. Notice that F takes dates 0 and 1 price levels as given but internalizes the impact of its bond issuance on the interest rate.

When selecting the "Sargent-Wallace" debt level, F optimally issues B_0 that will force M to set $P_1 = \underline{P}_1 + \alpha_M$, the largest date-1 price level that M prefers to default:

$$B_0 = (\underline{P}_1 + \alpha_M)(\bar{x} + \bar{b}) - R_0 X_0.$$

This course of action whereby F floods the bond market with paper so as to force M to "chicken out" and inflate away outstanding reserves at date 1 in order to ensure public solvency is closely related to that underlying the unpleasant monetarist arithmetic in Sargent and Wallace (1981). F creates a deficit that forces M to generate income in an inflationary way, simply by inflating away the value of reserves here.

Denoting $\tilde{b}(x_0)$ the real amount that F collects in the date-0 bond market when issuing the Sargent-Wallace debt level, Appendix B.1 shows that F's utility differential Δ between the "price-level taking" debt issuance (in which case $P_1 = \underline{P}_1$) and the "Sargent-Wallace" issuance (in which case $P_1 = \underline{P}_1 + \alpha_M$) can be expressed as:

$$\Delta = \underbrace{\hat{b}(x_0)(1-\beta r(1-x_0-\hat{b}(x_0))) - \tilde{b}(x_0)(1-\beta r(1-x_0-\tilde{b}(x_0)))}_{A} \tag{13}$$

$$-\underbrace{\beta R_0 X_0 \left(\frac{1}{\underline{P}_1} - \frac{1}{\underline{P}_1+\alpha_M}\right)}_{B}. \tag{14}$$

This latter expression of Δ illustrates the costs and benefits from the price-level taking issuance level versus the Sargent-Wallace one. Term A measures the difference in utility from allocating consumption over time in different ways across these actions. The sign of A is ambiguous as the allocation is suboptimal under the Sargent-Wallace debt level—the interest rate is too high relative to that in the price-level taking debt level since $\tilde{b}(x_0) \geq \hat{b}(x_0)$—but the total to be allocated is larger due to the lower value of reserves. Term B is positive. It is the benefit from eroding the value of reserves R_0X_0 with inflation.

The final step is the determination of the action of M in the date-0 market for reserves and the resulting choice of F between price-level taking and Sargent-Wallace debt levels. We focus here on the case in which all public liabilities are endogenously issued. Accordingly, we assume that the legacy reserves $R_{-1}X_{-1}$ are arbitrarily small—that is, always sufficiently small for our claims to hold given other parameter values.[11]

Proposition 3. ***(The determinants of monetary dominance)*** *Suppose $R_{-1}X_{-1}$ is sufficiently small. There exists a unique equilibrium. M does not issue new reserves at date 0, and so $X_0 = R_{-1}X_{-1}$. M thus has no income at date 0. Savers invest $x_0 = X_0/P_0^M$ in reserves and so the date-0 price level is P_0^M.*

- *If $\hat{g}_1(0) > 0$, F then issues the price-level taking debt level implying $P_1 = P_1^M$ is on target.*
- *If $\hat{g}_1(0) = 0$, F then issues the Sargent-Wallace debt level implying $P_1 = P_1^M + \alpha_M$.*

Proof. See Appendix B.1. □

In Sargent and Wallace (1981), the fiscal authority forces inflation by "moving first" and committing at the outset to a path of future deficits. Such a commitment is not

[11]We could simply assume $R_{-1}X_{-1} = 0$, but would then need an additional assumption (e.g., indivisibility) to ensure that the optimal reserve policy of M, that will consist in issuing an "arbitrarily small but strictly positive" X_0, has a well-defined solution.

possible in our setup. As a result, the fiscal authority has no choice but maximizing the current deficit to force a future inflation that generates income by reducing the real value of outstanding reserves. This implies that F must exhaust its borrowing capacity ($g_1 = 0$).

If the legacy reserves $R_{-1}X_{-1}$ are sufficiently small other things being equal, F finds this action unpalatable if $\hat{g}_1(0) > 0$. The Sargent-Wallace debt level entails in this case a finite suboptimal hike in the interest rate for an arbitrary small gain from inflating reserves away. The gain is in turn small because M can keep X_0 at a low level, thereby minimizing the base of the inflationary tax provoked by F at date 1. The only case in which the Sargent-Wallace debt level pays off and thus occurs is that in which F would be willing to borrow more at a higher rate had it more resources at date 1 ($\hat{g}_1(0) = 0$). The inflationary tax relaxes its constraint in this case.

Crowding out and monetary dominance. Proposition 3 states that the central bank fulfills its mandate despite being unable to commit and in the absence of any fiscal support provided $\hat{g}_1(0) > 0$. This occurs when the solution $\hat{b}(0)$ to the first-order condition associated with (9),

$$r(1-b) - br'(1-b) = \frac{1}{\beta}, \tag{15}$$

is such that $\hat{b}(0)r(1-\hat{b}(0)) < \bar{x} + \bar{b}$, which depends on $r(.)$, $\bar{x} + \bar{b}$, and β.

Accordingly, a sufficient condition for monetary dominance is that the Sargent-Wallace debt level entails a sufficiently large crowding out of private investment by public liabilities. Formally this occurs, other things being equal, if $r(.)$ is sufficiently steep (r' large in absolute terms). Holding the function $r(.)$ fixed, monetary dominance is also warranted when the future public resources $\bar{x} + \bar{b}$ are important all else equal, so that forcing M to chicken out requires draining large private savings out of private investments. Finally, and in connection with political-economy considerations, an impatient F (β small) is ceteris paribus more likely to be constrained, and thus to find the Sargent-Wallace issuance attractive. Notice that the risk neutrality of F stacks the deck in favor of fiscal dominance. In contrast, if F had strictly concave preferences, it would find the Sargent-Wallace debt level costly not only because it raises the marginal interest rate but also because it would shift its intertemporal marginal rate of substitution down and thus away from this marginal interest rate.

Fiscal requirements. It is worthwhile highlighting that when it enters into the Sargent-Wallace behavior, F does not derive any seigniorage income from it in equilibrium. Bonds and reserves are perfect substitutes in this setup and must earn the same equilibrium return. M anticipates a date-1 price above target in the announced rate R_0. As a result, if F could commit at the outset of the game to a fiscal requirement capping its nominal borrowing in the date-0 bond market, it would be happy to do so in order to tie its hands and avoid the Sargent-Wallace debt level. Proposition 4 below shows that this result need not hold when $R_{-1}X_{-1}$ is not arbitrarily small.

The case of a vanishingly small legacy liability $R_{-1}X_{-1}$ is a natural starting point in which all liabilities are essentially endogenous. The case in which $R_{-1}X_{-1}$ is finite is cumbersome in our general model. The following section tackles it in an alternative version of the model in which the return on private storage is constant.

3.2 The case of a constant return on private storage

Suppose that the return on private storage is a constant $r > 0$. This version of the model can be interpreted as a small open economy in which the public sector faces the world interest rate and the fiscal authority borrows in the local currency. Suppose also that:

$$\bar{x} + \bar{b} < r, \tag{16}$$

$$\frac{R_{-1}X_{-1}}{P_0^M} < \frac{\bar{x}}{r}. \tag{17}$$

Condition (16) rules out the unrealistic case in which the public sector can drain the whole savings in the economy.[12] Conditions (17) ensures that M need not be off target at date 0 because $R_{-1}X_{-1}$ is too large relative to its date-1 resources.

Proposition 4. ***(Fiscal and monetary dominance with a constant return on real storage)*** *There exists a unique equilibrium. Price levels are on target ($P_0 = P_0^M$*

[12]The assumption that $r \to_0 +\infty$ rules this out in the general model.

and $P_1 = P_1^M$) if and only if $\beta r > 1$ and

$$\frac{\bar{x}+\bar{b}}{r} \geq \left(\frac{\beta r - \frac{P_1^M}{P_1^M + \alpha_M}}{\beta r - 1} \right) \frac{R_{-1}X_{-1}}{P_0^M}. \tag{18}$$

Otherwise, if $\beta r \leq 1$, then $P_0 = P_0^M$, F issues the Sargent-Wallace debt level, and so $P_1 = P_1^M + \alpha_M$. If $\beta r > 1$ and (18) fails to hold, M selects depending on parameter values one of the three following options:

1. *Setting $P_0 = P_0^M$ and letting F issue at the Sargent-Wallace level so that $P_1 = P_1^M + \alpha_M$;*
2. *Setting $P_0 > P_0^M$ as the smallest value such that (18) holds when replacing P_0^M by P_0 so that F does not adopt the Sargent-Wallace debt level (and so $P_1 = P_1^M$);*
3. *Setting $P_0 = P_0^M$ and $R_0X_0 = P_1\bar{x} > R_{-1}X_{-1}$, where P_1 solves*

$$1 + \frac{\bar{b}}{\bar{x}} = \frac{\beta r - \frac{P_1}{P_1 + \alpha_M}}{\beta r - 1}, \tag{19}$$

   so as to commit to $P_1 > P_1^M$ at date 1, thereby discouraging F from issuing at the Sargent-Wallace level.

Proof. See Appendix B.2. □

Notice that under the assumption that $R_{-1}X_{-1}$ is arbitrarily small in the sense of Proposition 3, Proposition 4 implies that the central bank is independent if and only if $\beta r > 1$.

If $\beta r \leq 1$, the Sargent-Wallace debt level comes at no cost because F seeks to borrow as much as possible anyway. If the rate is higher, however, condition (18) shows that the central bank is independent if the present value of the resources of the public sector $(\bar{x}+\bar{b})/r$ is large relative to its current liabilities $R_{-1}X_{-1}/P_0^M$, multiplied by a coefficient larger than 1 that reflects the (off equilibrium path) gains from forcing fiscal dominance. In this case, the large inefficient borrowing required to induce M to chicken out brings too little gains. In other words, the central bank is independent if the public sector is "super solvent", or has a sufficiently large net wealth. The comparative statics properties with respect to public net wealth, measured for example by $[(\bar{x}+\bar{b})/r]/(R_{-1}X_{-1}/P_0^M)$, suggest that wealth fluctuations induce shifts between fiscal and monetary dominance. Such shifts will arise in the time series as public net wealth endogenously fluctuates in Section 3.4 in which $T > 1$.

Another interesting feature of the equilibrium arises when $\beta r > 1$ but $R_{-1}X_{-1}$ is too large and (18) fails to hold. Instead of just letting F embark on the Sargent-Wallace behavior, M may prefer to deter F from doing so by reducing the gains from inflating away reserves. This can be done either by raising P_0 above P_0^M so as to reduce the (real) base $R_{-1}X_{-1}/P_0$ to which the inflation rate applies, or by reducing the inflation rate α_M/P_1 itself by committing to a date-1 price $P_1 > P_1^M$.

Revisiting fiscal requirements. Proposition 4 states that there are three possible scenarii when monetary dominance does not hold: i) F issues at the Sargent-Wallace level; ii) M raises P_0 so that F issues at the price-taking level; iii) M raises P_1 so that F issues at the price-taking level. Expecting scenarii i) or iii), F would be happy to commit to a fiscal requirement at the outset if it could do so because it does not benefit ex-ante from inflation. Under scenario ii), by contrast, F and M disagree on fiscal requirements as F strictly benefits from inflating away legacy reserves.

Legacy debt. It is easy to accommodate for the presence of legacy debt B_{-1} due at date 0. In this case, if $\beta r > 1$, (18) ensuring monetary dominance becomes:

$$\frac{\bar{x} + \bar{b}}{r} \geq \frac{B_{-1}}{P_0^M} + \left(\frac{\beta r - \frac{P_1^M}{P_1^M + \alpha_M}}{\beta r - 1} \right) \frac{R_{-1}X_{-1}}{P_0^M}. \tag{20}$$

Expression (20) yields two insights. First, the coefficient that multiplies legacy reserves does not apply to legacy debt. The reason is that debt is due at date 0 whereas F can only generate fiscal dominance at date 1. The coefficient would apply if legacy debt was long term, due at date 1. Second, even though legacy debt cannot be inflated away, it still makes the Sargent-Wallace debt level relatively more appealing—by appearing on the RHS of (20)—because F needs to borrow to repay B_{-1} anyway even if $\beta r > 1$. The corresponding borrowing thus "comes for free" when issuing at the Sargent-Wallace level.

Return on central bank investments. Holding $\bar{b}$ and $R_{-1}X_{-1}$ fixed, monetary dominance is all the more likely because $\bar{x}$ is large. If one interprets $\bar{x}$ as including not only an exogenous demand for money but also the return on investments that M funded with the proceeds from issuing X_{-1} at date -1, then this implies that monetary dominance benefits from high expected return viewed from date 0. This shapes the risk-taking incentives of M when investing at date -1 given the net wealth of the government at this date. In particular, if fiscal dominance is very likely with safe instruments, M may be tempted to opt for assets with riskier returns to increase the probability of monetary dominance. Such gambling for resurrection behavior would parallel the behavior of investors subject to limited liability constraints as studied in the finance literature (see Allen and Gale, 2000, among other).

3.3 Why an independent central bank?

We directly assume for brevity the existence of a central bank with a price-stability mandate that has control over the nominal interest rate and over its balance sheet. It is however important to stress that such an institution is easy to motivate in our context. Suppose that a fiscal authority with preferences (1) is sole in charge of issuing both bonds and the unit of account (reserves). An interpretation of this situation is that F cannot even commit to let M operate its balance sheet independently.

Proposition 5. ***(Necessity of an independent central bank)*** *Suppose that F is in charge of the actions of M. Then the price level is infinite at all dates. F cannot issue any security at date 0.*

Proof. F sets $P_1 = +\infty$ in order to maximize its date-1 consumption and rational investors anticipating this do not invest at date 0, which implies $P_0 = +\infty$ as well. □

To be sure, this extreme result rests on the extreme (and unreasonable) assumption that hyperinflation comes at no exogenous cost for F whereas outright default does. Still, it is clear that F faces a standard commitment problem that would persist as long as some inflation is a more insidious way of generating income than outright default. We show that setting up an institution whose objective is to maintain the value of nominal claims may suffice to solve this commitment problem, even if this institution has no fiscal support nor any commitment ability itself. It may be sufficient that market forces such as the crowding out of private investment discourage the type of strategies envisioned by Sargent and Wallace (1981).

3.4 More periods: $T > 1$

This section extends the analysis to finite-horizon games such that $T \geq 2$. This offers interesting insights into the relationship between interest-rate levels and the dynamics of price levels. We still posit that $R_{-1}X_{-1}$ is arbitrarily small. In order to describe the equilibria, it is useful to introduce

$$b^* = \arg\max_{b\in[0,1)} \{b(1-\beta r(1-b))\}. \tag{21}$$

Notice that b^* coincides with $\hat{b}(0)$ defined in (9) in the case in which $\hat{g}_1(0) > 0$. For brevity we restrict the analysis to the case in which $b^* > 0$.

The equilibrium crucially depends on the position of $r(1-b^*)$ relative to 1. The case $r(1-b^*) < 1$ is the most relevant in the current context of "low rates".

Proposition 6. ***(Endogenous regime switching when*** $r(1-b^*) < 1$***)*** *Suppose* $r(1-b^*) < 1$. *There exists a unique equilibrium.* M *does not issue new reserves between dates 0 and* $T-1$.

There exists $\tau \in \{0; ...; T\}$ *such that for* $t \in \{0; ...; \tau\}$, $g_t > 0$ *and there is monetary dominance (*$P_t = P_t^M$*), whereas for* $t \in \{\tau+1; ...; T\}$ *(an empty set if* $\tau = T$*),* $g_t = 0$ *and there is fiscal dominance (*$P_t = P_t^M + \alpha_M$*).*

Proof. See Appendix B.3. □

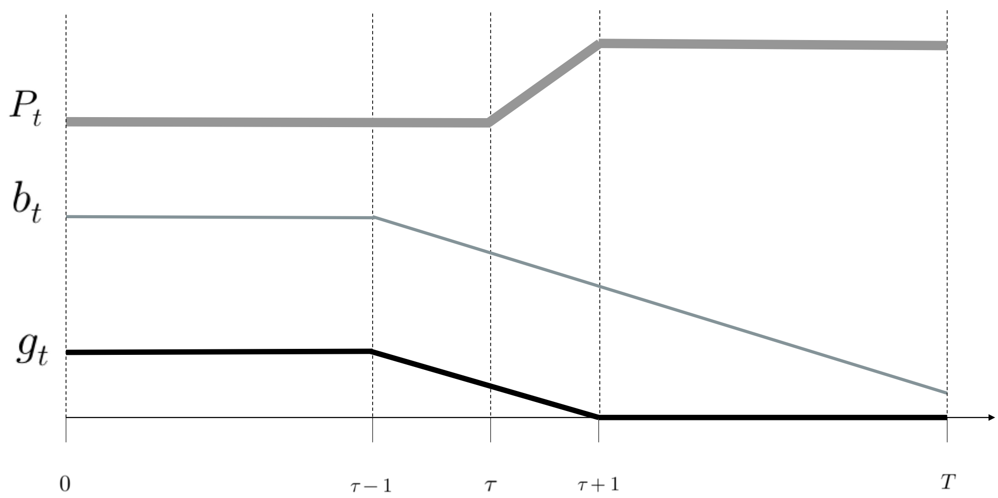

Figure 1: Dynamics of price level, debt, and deficit.

Figure 1 illustrates the generic dynamics[13] of the price level, debt, and deficit. All variables are constant up to $\tau - 1$, and $P_t = P_t^M$. F can borrow the optimal level b^* and consume since $b^*(1 - r(1 - b^*)) > 0$. Then at date τ debt and consumption tank. At $\tau + 1$, the price level jumps to the fiscal-dominance level $P_{\tau+1}^M + \alpha_M$ and F uses its debt issuance to roll over legacy debt, thereby no longer consuming until the terminal date.

The reason why regime switches this way is that the terminal resources $\bar{x} + \bar{b}$ discounted at $r(1 - b^*) < 1$ become large at the initial dates, and thus F faces no borrowing constraint at these dates. As the terminal date gets closer, a financial constraint may start binding, and F may as well adopt the Sargent-Wallace debt level from then on. Regime switching contrasts with rules-based model (e.g., Leeper, 1991), in which the perfect-foresight expectation of future fiscal (or monetary) dominance generates immediate fiscal (or monetary) dominance.

In the current US context of low rates, large deficits and outstanding amounts of public liabilities, yet stable price levels, this latter equilibrium in which a similar situation prevails for a possibly arbitrary long time until it ultimately morphs into one of a constrained public sector and inflation is interesting.

The case $r(1 - b^*) \geq 1$ is more involved as F may find itself forced to roll over a level of legacy debt that is larger than the ex-post optimum. Generically, dominance switches from fiscal to monetary when $r(1 - b^*) \geq 1$—in the opposite direction from that when $r(1 - b^*) < 1$. Interestingly, unlike when $T = 1$, fiscal dominance may initially prevail even though F does not borrow against its entire terminal resources ($g_T > 0$). Appendix D offers a detailed treatment of this case $r(1 - b^*) \geq 1$.

4 Infinite horizon

We now turn to the infinite-horizon version of the model. This entails two significant departures from the economies studied thus far. First, the public sector cannot back reserves and bonds with real resources $\bar{x} + \bar{b}$. Public liabilities are therefore pure bubbles. Second, the private sector can enter into strategies that grant it significantly much more influence over fiscal and monetary policies than in the finite-horizon setting.

Both the possibility of bubbles and that of potentially complex history-dependent strategies create room for a plethora of equilibria. There are many possible bubbly

[13] By generic we mean for parameter values such that $1 \leq \tau \leq T - 1$.

paths, and this multiplicity creates in turn room for strategies whereby the bubbly path on which savers coordinate going forward is history dependent. The goal of this section is to exhibit a non-trivial equilibrium in which both F and M can collect resources, and to show that the off-equilibrium-path behavior of the private sector is the true determinant of the price level in this equilibrium.

We suppose in this section that $r(1) < 1$, a necessary and sufficient condition for bubbles to exist. We also suppose that b^* defined in (21) is strictly positive.

Consider a series of strictly positive numbers $(\bar{x}_t, \bar{b}_t)_{t\geq 0}$ such that:

$$\bar{x}_0 > \frac{R_{-1}X_{-1}}{P_0^M}, \tag{22}$$

and for all $t \geq 0$

$$\bar{x}_t + \bar{b}_t < 1, \tag{23}$$

$$\bar{x}_{t+1} > r(1 - \bar{b}_t - \bar{x}_t)\bar{x}_t, \tag{24}$$

$$\bar{b}_{t+1} + \bar{x}_{t+1} = r(1 - \bar{b}_t - \bar{x}_t)(\bar{b}_t + \bar{x}_t). \tag{25}$$

Given that $r(1) < 1$, such a series exists if $R_{-1}X_{-1}/P_0^M$ is sufficiently small, which we assume.

Proposition 7. ***(Market discipline may enforce monetary dominance)*** *Suppose* $(\bar{x}_t, \bar{b}_t)_{t\in\mathbb{N}}$ *admits a sufficiently small upper bound.*

- ***Fiscal-dominance equilibrium****. There exists an equilibrium in which the price level is* $P_t = P_t^M + \mathbb{1}_{\{t>0\}}\alpha_M$. *No new reserves are issued. The public sector collects* $\bar{b}_t + \bar{x}_t$ *at every date* t*.* F *consumes at date 0 and rolls over debt afterwards.*

- ***Monetary-dominance equilibrium.*** *There also exists an equilibrium in which the price level is* $P_t = P_t^M$. *No new reserves are issued. The public sector collects* $\bar{b}_t + \bar{x}_t$ *at every date* t*.* F *consumes at date 0 and rolls over debt afterwards.*

Proof. See Appendix B.4. □

The fiscal-dominance equilibrium can be viewed as the infinite-horizon extension of a finite-horizon equilibrium in which F is constrained at each date t because x_{t+1} and

b_{t+1} are sufficiently small, and so it may as well issue at the Sargent-Wallace level. The monetary-dominance equilibrium features the exact same real quantities and utility of F as the fiscal-dominance one. The price level is however on target, an outcome that would be out of reach under finite horizon given a constrained fiscal authority.

A difference in savers' strategy profiles across these two equilibria suffices to induce this difference in price levels. As detailed in the proof of Proposition 7, in the fiscal-dominance equilibrium, savers are purely forward-looking. Their investment decisions are only based on expected returns given history and strategy profiles. Savers' beliefs about future demand for public securities are self-fulfilling as they define the largest possible bubbles that F and M can generate by issuing securities, and that F and M do optimally generate in equilibrium.

The monetary-dominance equilibrium adds the feature that if she observes a price level $P_{t-1} \neq P_{t-1}^{M}$, then a saver shuns the reserve market at date t. In other words, savers prick the bubble on reserves if the central bank has missed its target in the past. This does not occur along the equilibrium path but gives commitment power to M, because any attempt at slightly inflating away public liabilities to avoid default would result in the economy embarking on autarky and in the inability of the public sector to issue any nominal claims. M thus prefers to default and anticipating this, F avoids Sargent-Wallace issuances. Another way of saying this is that market discipline creates an endogenous value of α_M equal to 0.

In sum, when an important component of public liabilities is bubbly, there is room for "market dominance": The market has the possibility to "move first" and determine the price level by exploiting the multiplicity of bubbly paths. In fact, it is easy to see that the market could enforce any price level, regardless of the objectives of F and M, as long as autarky minimizes their utilities. It is important to stress that such market discipline would be effective even if the public sector could partially back its liabilities with a stream of future resources. All that matters for this result to hold is that a sufficiently large fraction of the liquidity supplied by the public sector is a self-fulfilling phenomenon that the private sector can credibly make history-dependent. If, on the other hand, a version of the model with infinite horizon and a stream of future resources displayed dynamic efficiency, then the equilibrium would be unique, and this would rule out market dominance.

Unsurprisingly, this market discipline closely relates to that in the sovereign-default literature pioneered by Eaton and Gersovitz (1981), as inflation is a particular form of default. It also relates to the literature that explores market discipline as a device to enforce fiscal rules (Halac and Yared, 2017, e.g.).

5 Concluding remarks

This paper solves a full-fledged model of strategic dynamic interactions between fiscal and monetary authorities with conflicting objectives. Its main goal is to identify which primitives of the economy determine whether the regime is one of fiscal or monetary dominance. We find that a monetary authority that lacks both commitment power and fiscal support may still be in the position of imposing its objectives if market forces make the inflationary fiscal expansion envisioned by Sargent and Wallace unpalatable to the fiscal authority. This is so when the market responds to large debt issuances with a high required (real) rate.

We believe that our setting opens many avenues for future research. Notably, a number of assumptions that seem natural for this first pass could be relaxed. In particular, we focus on the case in which public liabilities are perfect substitutes, public debt is only short-term, and prices are flexible. If the liabilities of the central bank provided superior liquidity services, this would boost its ability to generate public revenue, thereby possibly exacerbating the conflict between fiscal and monetary objectives. The possibility of long-term debt may give further tools for the government to finely tailor the timing of its future repayments in order to manage in turn the timing of the monetary authority chickening out. On the other hand, the monetary authority may also alter the maturity structure of overall public liabilities by trading government bonds against reserves. Also, some price rigidity would grant the monetary authority the ability to manipulate real rates and output, and this would in turn generate extra incentives for the fiscal authority to lead the monetary authority to chicken out. Finally, additional natural routes for future research include the introduction of shocks to public resources or/and government preferences, and that of multiple non-cooperative fiscal authorities.

References

Aiyagari, S. R. and E. R. McGrattan (1998): "The optimum quantity of debt," *Journal of Monetary Economics*, 42, 447–469.

Alesina, A. (1987): "Macroeconomic policy in a two-party system as a repeated game," *Quarterly Journal of Economics*, 102, 651–678.

Alesina, A. and G. Tabellini (1987): "Rules and discretion with noncoordinated monetary and fiscal policies," *Economic Inquiry*, 25, 619–630.

Allen, F. and D. Gale (2000): "Bubbles and crises," *Economic Journal*, 110, 236–255.

Alvarez, F., P. J. Kehoe, and P. A. Neumeyer (2004): "The Time Consistency of Optimal Monetary and Fiscal Policies," *Econometrica*, 72, 541–567.

Angeletos, G.-M., F. Collard, and H. Dellas (2020): "Public Debt as Private Liquidity: Optimal Policy," CEPR Discussion Papers 15488, C.E.P.R. Discussion Papers.

Asriyan, V., L. Fornaro, A. MartÃn, and J. Ventura (2019): "Monetary Policy for a Bubbly World," CEPR Discussion Papers 13803, C.E.P.R. Discussion Papers.

Azzimonti, M. and P. Yared (2019): "The optimal public and private provision of safe assets," *Journal of Monetary Economics*, 102, 126–144.

Barro, R. J. and D. B. Gordon (1983a): "A Positive Theory of Monetary Policy in a Natural Rate Model," *Journal of Political Economy*, 91, 589–610.

——— (1983b): "Rules, discretion and reputation in a model of monetary policy," *Journal of monetary economics*, 12, 101–121.

Barthélemy, J., E. Mengus, and G. Plantin (2020): "Public Liquidity Demand and Central Bank Independence," CEPR Discussion Paper 14160.

Barthélemy, J. and G. Plantin (2018): "Fiscal and Monetary Regimes: A Strategic Approach," CEPR Discussion Paper 12903.

Bassetto, M. (2002): "A Game-Theoretic View of the Fiscal Theory of the Price Level," *Econometrica*, 70, 2167–2195.

Bassetto, M. and W. Cui (2018): "The fiscal theory of the price level in a world of low interest rates," *Journal of Economic Dynamics and Control*, 89, 5–22.

Bassetto, M. and T. Messer (2013): "Fiscal Consequences of Paying Interest on Reserves," *Fiscal Studies*, 34, 413–436.

Bassetto, M. and T. J. Sargent (2020): "Shotgun Wedding: Fiscal and Monetary Policy," Working Paper 27004, National Bureau of Economic Research.

Benigno, P. (forthcoming): "A Central Bank Theory of Price Level Determination," *American Economic Journal: Macroeconomics*.

Bianchi, F., R. Faccini, and L. Melosi (2020): "Monetary and Fiscal Policies in Times of Large Debt: Unity is Strength (REVISED May 2020)," Working Paper Series WP 2020-13, Federal Reserve Bank of Chicago.

Bianchi, F., H. Kung, and T. Kind (2019): "Threats to Central Bank Independence: High-Frequency Identification with Twitter," NBER Working Papers 26308, National Bureau of Economic Research, Inc.

Brunnermeier, M. K., S. A. Merkel, and Y. Sannikov (2020): "The Fiscal Theory of Price Level with a Bubble," Working Paper 27116, National Bureau of Economic Research.

Buiter, W. H. (2002): "The Fiscal Theory of the Price Level: A Critique," *Economic Journal*, 112, 459–480.

Calvo, G. (1978): "On the Time Consistency of Optimal Policy in a Monetary Economy," *Econometrica*, 46, 1411–28.

Chari, V. V. and P. J. Kehoe (1990): "Sustainable Plans," *Journal of Political Economy*, 98, 783–802.

Cochrane, J. H. (2001): "Long-Term Debt and Optimal Policy in the Fiscal Theory of the Price Level," *Econometrica*, 69, 69–116.

——— (2005): "Money as Stock," *Journal of Monetary Economics*, 52, 501–528.

Coibion, O., Y. Gorodnichenko, and M. Weber (2021): "Fiscal Policy and Households' Inflation Expectations: Evidence from a Randomized Control Trial," Tech. rep., National Bureau of Economic Research.

Diamond, P. A. (1965): "National Debt in a Neoclassical Growth Model," *American Economic Review*, 55, 1126–1150.

Dixit, A. and L. Lambertini (2003): "Interactions of commitment and discretion in monetary and fiscal policies," *American Economic Review*, 93, 1522–1542.

Eaton, J. and M. Gersovitz (1981): "Debt with Potential Repudiation: Theoretical and Empirical Analysis," *Review of Economic Studies*, 48, 289–309.

Galí, J. (2014): "Monetary Policy and Rational Asset Price Bubbles," *American Economic Review*, 104, 721–52.

Gorton, G. and G. Ordonez (2021): "The Supply and Demand for Safe Assets," Mimeo.

Halac, M. and P. Yared (2017): "Fiscal Rules and Discretion under Self-Enforcement," NBER Working Papers 23919, National Bureau of Economic Research, Inc.

——— (2020): "Inflation Targeting under Political Pressure," *Independence, Credibility, and Communication of Central Banking, ed. by E. Pastén and R. Reis, Santiago, Chile, Central Bank of Chile*, 7–27.

Hall, R. E. and R. Reis (2015): "Maintaining Central-Bank Financial Stability under New-Style Central Banking," CEPR Discussion Papers 10741, C.E.P.R. Discussion Papers.

Holmström, B. and J. Tirole (1998): "Private and Public Supply of Liquidity," *Journal of Political Economy*, 106, 1–40.

Jacobson, M. M., E. M. Leeper, and B. Preston (2019): "Recovery of 1933," NBER Working Papers 25629, National Bureau of Economic Research, Inc.

Krishnamurthy, A. and A. Vissing-Jorgensen (2012): "The Aggregate Demand for Treasury Debt," *Journal of Political Economy*, 120, 233–267.

Leeper, E. M. (1991): "Equilibria under 'active' and 'passive' monetary and fiscal policies," *Journal of Monetary Economics*, 27, 129–147.

Ljungqvist, L. and T. J. Sargent (2018): *Recursive Macroeconomic Theory, Fourth Edition*, no. 0262038668 in MIT Press Books, The MIT Press.

Lucas, R. J. and N. L. Stokey (1983): "Optimal fiscal and monetary policy in an economy without capital," *Journal of Monetary Economics*, 12, 55–93.

Martin, F. M. (2015): "Debt, inflation and central bank independence," *European Economic Review*, 79, 129 – 150.

McCallum, B. T. (2001): "Indeterminacy, bubbles, and the fiscal theory of price level determination," *Journal of Monetary Economics*, 47, 19–30.

Niepelt, D. (2004): "The Fiscal Myth of the Price Level," *Quarterly Journal of Economics*, 119, 277–300.

Persson, M., T. Persson, and L. E. Svensson (2006): "Time consistency of fiscal and monetary policy: a solution," *Econometrica*, 74, 193–212.

Sargent, T. J. and N. Wallace (1981): "Some unpleasant monetarist arithmetic," *Quarterly Review*.

Sims, C. A. (1994): "A Simple Model for Study of the Determination of the Price Level and the Interaction of Monetary and Fiscal Policy," *Economic Theory*, 4, 381–399.

——— (2003): "Fiscal Aspects of Central Bank Independence," Princeton University.

Stokey, N. L. (1991): "Credible public policy," *Journal of Economic Dynamics and Control*, 15, 627–656.

Tabellini, G. (1986): "Money, debt and deficits in a dynamic game," *Journal of Economic Dynamics and Control*, 10, 427–442.

Woodford, M. (1990): "Public Debt as Private Liquidity," *American Economic Review*, 80, 382–388.

——— (1994): "Monetary policy and price level determinacy in a cash-in-advance economy," *Economic theory*, 4, 345–380.

——— (1995): "Price-level determinacy without control of a monetary aggregate," *Carnegie-Rochester Conference Series on Public Policy*, 43, 1–46.

——— (2001): "Fiscal Requirements for Price Stability," *Journal of Money, Credit and Banking*, 33, 669–728.

Appendix

A Perfect-foresight competitive equilibria

Fix $t \in \mathbb{N}$ and $(\mathcal{E}_s)_{s<t}$ a given history with $B_{-1} = b^M_{-1} = X_{-1} = 0$. A competitive equilibrium from date t on given history $(\mathcal{E}_s)_{s<t}$ is a sequence $(\mathcal{E}_s)_{s\geq t}$ such that at every date $s \geq t$:

- $X_s \geq X_{s-1}R_{s-1}$, $B_s \geq 0$, $b^M_s \geq 0$.
- Budget constraints hold:

$$X_s - R_{s-1}X_{s-1} + \frac{(1-l_s)b^M_{s-1}P_{s-1}}{Q_{s-1}} = P_s(\theta_s + b^M_s), \tag{26}$$

$$Q_sB_s - (1-l_s)B_{s-1} + P_s\theta_s = P_sg_s. \tag{27}$$

- Markets clear:

$$X_s = P_sx_s, \tag{28}$$

$$Q_sB_s = P_s(b_s + b^M_s). \tag{29}$$

- Savers optimize:

$$(x_s, b_s) \in \arg\max_{(x,b)\in[0,1]^2} \left\{ \frac{R_sP_sx}{P_{s+1}} + \frac{(1-l_{s+1})P_sb}{Q_sP_{s+1}} + f(1-x-b) \right\} \tag{30}$$

$$s.t.\ x + b \leq 1.$$

Condition (26) is the flow budget constraint of M and (27) that of F, (28) is the reserve-market clearing condition, and (29) that of the bond market.

In the absence of any fiscal backing, a necessary and sufficient condition for the existence of equilibria in which both M and F issue nonnegative quantities of liabilities is that $r(1) < 1$. In this case, F and M can issue bubbles—unbacked liabilities that can repay themselves. Consider for example the following steady state corresponding to a given fixed price level $P > 0$ and to the largest possible total demand for public liquidity. Let $(x, b) \in (0, 1)^2$ such that $r(1 - x - b) = 1$. There exists a steady state in which savers bid x for reserves and b for bonds at each date. The price level is P. M issues xP reserves at date 0 and then none so that $X_t = Px$, and announces an interest rate $R_t = 1$. F issues $B_t = bP$ bonds at each date and the bond price is $Q_t = 1$. M pays an initial dividend equal to x to F who consumes $x + b$ at date 0 and then nothing.

B Proofs

B.1 Proof of Lemma 1, Lemma 2 and Proposition 3

This section solves for the whole two-dates game using backwards induction in order to prove the proposition and the lemmata along the way.

Second stage of date 1. If F chooses a given haircut l_1, it receives from M

$$\theta_1(l_1) = \bar{x} - \frac{R_0 X_0}{P_1} + \frac{(1-l_1) b_0^M P_0}{Q_0 P_1}. \tag{31}$$

It is therefore optimal for F to set $l_1 = 0$ if

$$g_1 = \bar{b} + \theta_1(0) - \frac{B_0}{P_1} = \bar{x} + \bar{b} - \frac{B_0 - \frac{b_0^M P_0}{Q_0}}{P_1} - \frac{R_0 X_0}{P_1} \geq 0 \tag{32}$$

and $l_1 = 1$ otherwise, in which case F consumes $g_1 = \bar{x} + \bar{b} - R_0 X_0 / P_1$.

First stage of date 1. Here M can set the price at any level $P_1 \geq R_0 X_0 / \bar{x}$ by issuing $X_1 - R_0 X_0 \geq 0$ such that $X_1 = P_1 \bar{x}$. So, if the smallest price level that ensures that the net liabilities of the public sector are covered by its resources is too large:

$$B_0 - b_0^M P_0 / Q_0 + R_0 X_0 > (\bar{x} + \bar{b}) \left(\max \left\{ P_1^M ; \frac{R_0 X_0}{\bar{x}} \right\} + \alpha_M \right), \tag{33}$$

M prefers to force default and sets $P_1 = \max\{P_1^M ; R_0 X_0 / \bar{x}\}$. Otherwise, M averts default by setting

$$P_1 = \max \left\{ P_1^M ; \frac{R_0 X_0}{\bar{x}} ; P_1^F \right\}, \tag{34}$$

where

$$P_1^F = \frac{B_0 - \frac{b_0^M P_0}{Q_0} + R_0 X_0}{\bar{x} + \bar{b}}. \tag{35}$$

This proves Lemma 1. Anticipating date 1 as above, agents play date 0 as follows.

Third stage of date 0. M simply transfers $x_0 - R_{-1}X_{-1}/P_0 - b_0^M$ to F who consumes it on top of the amount $b_0 + b_0^M$ collected in the bond market. F thus consumes $x_0 + b_0 - R_{-1}X_{-1}/P_0$, independent of b_0^M.

Second stage of date 0. Suppose that F issues $B_0 > 0$ bonds. There cannot be default at date 1: Since $l_1 = 1$ in case of default from above, savers' optimality implies $b_0 = 0$ in this case, and F only receives b_0^M from M in the bond market against an empty promise. But then F would be strictly better off not issuing bonds ($B_0 = 0$) and receiving b_0^M as a transfer from M at stage 3 of date 0, as this averts default leaving g_0 and g_1 unchanged. Furthermore, it must be that $b_0 > 0$. Otherwise F might as well not issue bonds and receive b_0^M as a dividend again since it would not affect neither price nor consumption levels. That it does not affect the date-1 price level stems from the fact that P_1^F depends only on $B_0 - b_0^M P_0/Q_0$.

Market clearing in the bond market reads:

$$Q_0 B_0 = P_0(b_0 + b_0^M), \tag{36}$$

and savers' rationality implies $b_0 + x_0 < 1$ and

$$\frac{P_0}{P_1 Q_0} = r(1 - b_0 - x_0). \tag{37}$$

Given the above determination of P_1 by (34), relations (36) and (37) form a system in (b_0, Q_0) given $(h_0, R_0, X_0, x_0, B_0, b_0^M)$ that has a unique solution.

To solve for the choice of B_0 by F given history, we proceed in two steps. A given issuance B_0 leads from above either to $P_1 = P_1^F$ or $P_1 > P_1^F$. We solve for the optimal action of F conditionally on each outcome. We then compare F's utility in each case in order to derive the unconditionally optimal action.

Case 1: Optimal B_0 if the solvency condition is not binding at date 1. Suppose first that F selects B_0 such that the continuation of the game satisfies $P_1 > P_1^F$. In this case one can without loss of generality replace B_0 with $B_0 - P_0 b_0^M / Q_0$ and assume $b_0^M = 0$ since this does not affect price nor consumption levels. Combining (36) and (37) yields

$$\frac{B_0}{P_1} = r(1 - x_0 - b_0)b_0. \tag{38}$$

which shows in turn that F by selecting B_0 decides on the real amount b_0 to borrow at the rate $r(1 - b_0 - x_0)$ taking P_1 as given. It must therefore be that

$$b_0 = \hat{b}(x_0) = \arg\max_b \{g_0 + \beta g_1\} \tag{39}$$

s.t.

$$g_0 = x_0 + b - \frac{R_{-1}X_{-1}}{P_0}, \tag{40}$$

$$g_1 = \bar{x} + \bar{b} - \frac{R_0 X_0}{P_1} - r(1 - x_0 - b)b, \tag{41}$$

$$0 \leq b < 1 - x_0, 0 \leq g_1. \tag{42}$$

Claim 1. $\hat{b}(x_0)$ *is unique and such that* $\hat{b}(x_0) + x_0$ *continuously (weakly) increases w.r.t.* x_0*.*

Proof. The function $b \mapsto b(1 - \beta r(1 - x_0 - b))$ is concave and thus admits a unique maximum over $[0, 1 - x_0)$ since it tends to $-\infty$ at $1 - x_0$, and continuity stems from the continuity of the objective and constraints. The first-order condition reads:

$$r(1 - x_0 - b) - r'(1 - x_0 - b)b = \frac{1}{\beta}. \tag{43}$$

Both functions $r(1 - x_0 - b)$ and $-r'(1 - x_0 - b)b$ on the LHS are increasing in x_0, b, and so b must decrease and $x_0 + b$ increase if x_0 increases. □

Case 2: Optimal B_0 if the solvency condition is binding at date 1. Suppose now that F selects B_0 such that the continuation of the game satisfies $P_1 = P_1^F$. Plugging

(37) in (35) yields

$$P_1^F = \frac{R_0 X_0 + B_0}{\bar{x} + \bar{b} + r(1 - b_0 - x_0) b_0^M}, \tag{44}$$

showing that M optimally sets $b_0^M = x_0 - R_{-1}X_{-1}/P_0$ to minimize $P_1 = P_1^F$. Combining (36) and (37) with the above equation to eliminate P_1^F yields

$$b_0 = \frac{B_0(\bar{x} + \bar{b})}{(B_0 + R_0 X_0) r(1 - b_0 - x_0)} - \frac{(x_0 - \frac{R_{-1}X_{-1}}{P_0}) R_0 X_0}{B_0 + R_0 X_0}. \tag{45}$$

Simple algebra shows that this implies that B_0 increases with respect to b_0. F thus chooses the maximum B_0 that is compatible with absence of default, that is, B_0 such that

$$P_1 = \underline{P}_1 + \alpha_M \tag{46}$$

where

$$\underline{P}_1 \equiv \max \left\{ P_1^M; \frac{R_0 X_0}{\bar{x}} \right\}. \tag{47}$$

Combining again (35), (36), and (37) yields that the value $\tilde{b}_0(x_0)$ leading to this solves:

$$\tilde{b}(x_0) = \frac{1}{r(1 - x_0 - \tilde{b}(x_0))} \left(\bar{x} + \bar{b} - \frac{R_0 X_0}{\underline{P}_1 + \alpha_M} \right). \tag{48}$$

As a result, F's utility differential Δ between the "price-level taking" debt level (such that $P_1 = \underline{P}_1$) and the "Sargent-Wallace" debt level (such that $P_1 = \underline{P}_1 + \alpha_M$) is:

$$\Delta = x_0 - R_{-1}X_{-1}/P_0 + \hat{b}(x_0) + \beta \left(\bar{x} + \bar{b} - r(1 - x_0 - \hat{b}(x_0)) \hat{b}(x_0) - \frac{R_0 X_0}{\underline{P}_1} \right) \tag{49}$$

$$- (x_0 - R_{-1}X_{-1}/P_0 + \tilde{b}(x_0)) \tag{50}$$

$$= \underbrace{\hat{b}(x_0)[1 - \beta r(1 - x_0 - \hat{b}(x_0))] - \tilde{b}(x_0)(1 - \beta r(1 - x_0 - \tilde{b}(x_0))}_{A} \tag{51}$$

$$- \underbrace{\beta R_0 X_0 \left(\frac{1}{\underline{P}_1} - \frac{1}{\underline{P}_1 + \alpha_M} \right)}_{B}. \tag{52}$$

This latter expression of Δ illustrates the costs and benefits from the price-level taking issuance versus the Sargent-Wallace issuance. Term A measures the difference in utility from allocating consumption over time in different ways across debt levels. The sign of A is ambiguous as the allocation is suboptimal under the Sargent-Wallace issuance but the total to be allocated is larger due to the lower value of reserves. Term B is positive. It is the benefit from eroding the value of reserves R_0X_0 with inflation.

This proves Lemma 2. As now shown by the analysis of the initial stage—the date-0 reserve market, this tradeoff between distorting consumption and inflating outstanding reserves away shapes the equilibrium.

First stage of date 0. Suppose that $R_{-1}X_{-1}$ is arbitrarily small. Market clearing in the reserve market reads:

$$X_0 = P_0 x_0, \tag{53}$$

and savers' rationality implies

$$\frac{R_0 P_0}{P_1} = r(1 - b_0 - x_0). \tag{54}$$

Given the continuation of the game derived above, relations (53) and (54) form a system in (x_0, P_0) as a function of (h_0, X_0, R_0) with a unique solution.

Suppose first that $\hat{b}(0)$ is strictly smaller than $(\bar{x}+\bar{b})/r(1-\hat{b}(0))$. In this case, M sets $X_0 = R_{-1}X_{-1}$ and announces $R_0 = r(1-X_0/P_0^M - \hat{b}(X_0/P_0^M))P_1^M/P_0^M$. This corresponds to an equilibrium in which savers invest X_0/P_0^M in the market for reserves and $\hat{b}(X_0/P_0^M)$ in that for bonds, and the price level is on M's target at each date. The reason is that for $R_{-1}X_{-1}$ sufficiently small, $\hat{b}(X_0/P_0^M)$ is interior as it converges to $\hat{b}(0)$, and so A is positive, bounded away from 0, whereas the gains B are sufficiently small. In particular, the lexicographic preferences of M imply that minimizing x_0 this way is optimal because this minimizes the distortions in F's choice of b given that prices are on target. This is the only reason these lexicographic preferences play a role.

Suppose then that $\hat{b}(0) = (\bar{x}+\bar{b})/r(1-\hat{b}(0)$. In this case, it is always optimal for F to issue at the Sargent-Wallace level in the bond market since A is always (weakly) negative: The increase in date-1 resources induced by the lower value of reserves in the Sargent-Wallace debt level relaxes the binding constraint $g_1 \geq 0$ in the consumptionsmoothing one. As a result, $\underline{P}_1 + \alpha_M$ is the lowest price that M can hope for at date 1. Since the largest one that it prefers to default is $P_1^M + \alpha_M$, this has to be the date-1 price. Accordingly, M sets $X_0 = R_{-1}X_{-1}$ and announces $R_0 = r(1 - X_0/P_0^M - \tilde{b}(X_0/P_0^M))(P_1^M + \alpha_M)/P_0^M$. This corresponds to an equilibrium in which savers invest X_0/P_0^M in the market for reserves and $\tilde{b}(X_0/P_0^M) = \hat{b}(X_0/P_0^M)$ in that for bonds, and the price levels are $P_0 = P_0^M$ and $P_1 = P_1^M + \alpha_M$.

This proves Proposition 3.

We have established that $X_0 = R_{-1}X_{-1}$ is strictly optimal in the case in which $\hat{b}(0)(1 - r(\hat{b}(0)) < \bar{x} + \bar{b}$ because of M's assumed lexicographic preferences. Conversely, when $\hat{b}(0) = (\bar{x}+\bar{b})/r(1-\hat{b}(0))$, larger reserves would not affect price levels nor F's utility since the public sector exhausts its aggregate borrowing capacity anyway. In this case, we select the equilibrium in which $X_0 = R_{-1}X_{-1}$ because this is the one that minimizes F's (ex-post) gains from the Sargent-Wallace debt level as it minimizes the base to which it applies. This would be the unique equilibrium if F was drawing some arbitrarily small costs from issuing bonds once the bond market opens up.

B.2 Proof of Proposition 4

Viewed from the stage of the date-0 bond market, the Sargent-Wallace debt level grants F a utility

$$x_0 - \frac{R_{-1}X_{-1}}{P_0} + \frac{1}{r}\left(\bar{x} + \bar{b} - \frac{R_0 X_0}{\underline{P}_1 + \alpha_M}\right), \tag{55}$$

where $\underline{P}_1$ is defined in (47), whereas the price-level taking debt level yields

$$x_0 - \frac{R_{-1}X_{-1}}{P_0} + \hat{b}(x_0) + \beta\left(\bar{x} + \bar{b} - r\hat{b}(x_0) - \frac{R_0 X_0}{\underline{P}_1}\right). \tag{56}$$

When $\beta r \leq 1$, $\hat{b}(x_0) = (\bar{x} + \bar{b} - R_0 X_0/\underline{P}_1)/r$, otherwise, $\hat{b}(x_0) = 0$. The Sargent-Wallace borrowing clearly dominates the price-level taking one if $\beta r \leq 1$. If $\beta r > 1$, the Sargent-Wallace behavior is (weakly) dominated if

$$\frac{\bar{x} + \bar{b}}{r} \geq \frac{\beta r - \frac{\underline{P}_1}{\underline{P}_1 + \alpha_M}}{\beta r - 1}\frac{R_0 X_0}{r\underline{P}_1}. \tag{57}$$

Going backward to the date-0 reserve market, this implies that if $\beta r > 1$ and the above expression holds with $X_0 = x_0 P_0^M = R_{-1}X_{-1}$, $\underline{P}_1 = P_1^M$ and $R_0 = rP_1^M/P_0^M$ then the equilibrium features the price-level taking debt level. The above expression is in this case (18):

$$\frac{\bar{x}+\bar{b}}{r} \geq \left(\frac{\beta r - \frac{P_1^M}{P_1^M+\alpha_M}}{\beta r - 1} \right) \frac{R_{-1}X_{-1}}{P_0^M}.$$

If $\beta r > 1$ and (18) fails to hold, M selects depending on parameter values one of the three following options. First, it can set $P_0 = P_0^M$ and let F issue the Sargent-Wallace debt level so that $P_1 = P_1^M + \alpha_M$.

Second, it can set $P > P_0^M$ as the smallest value such that (18) holds when substituting P_0^M with P. More precisely, M sets $X_0 = R_{-1}X_{-1}$, $P_1 = P_1^M$, and $R_0 = rP_1^M/P$. This way, F does not enter into the Sargent-Wallace behavior.

Finally, it can commit to the lowest date-1 price level $P' > P_1^M$ such that (18) holds when substituting P_1^M with P'. Such a P' solves

$$1 + \frac{\bar{b}}{\bar{x}} = \frac{\beta r - \frac{P'}{P'+\alpha_M}}{\beta r - 1}. \tag{58}$$

More precisely, M announces $X_0 = P_0^M \bar{x}/r$, and $R_0 = rP'/P_0^M$, so that $P_0 = P_0^M$, $x_0 = \bar{x}/r$, and $P_1 = P'$. The intuition why this price level discourages the Sargent-Wallace debt level is that it generates a sufficiently low inflation rate α_M/P.

B.3 Proof of Proposition 6

Consider the T-dates version of F's optimization program (9):

$$(b_0; ...; b_{T-1}) = \arg\max \left\{ \sum_{t=0}^{T} \beta^t g_t \right\} \tag{59}$$

s.t.

$$0 \leq g_0 = b_0, \tag{60}$$

$$0 \leq g_{t+1} = b_{t+1} - r(1-b_t)b_t, \forall 0 \leq t \leq T-2 \tag{61}$$

$$0 \leq g_T = \bar{x} + \bar{b} - r(1-b_{T-1})b_{T-1}. \tag{62}$$

Claim. When $r(1-b^*) < 1$, the solution $(b_t)_{t\in\{0;...;T-1\}}$ to program (59) is decreasing and bounded above by b^*. There exists $\tau \in \{0;...;T\}$ such that $g_t > 0$ over $\{0;...;\tau\}$ and $g_t = 0$ for $t \geq \tau+1$. (The latter set is empty if $\tau = T$.) Furthermore, $b_t = b^*$ for $t \in \{0;...;\tau-1\}$ if $\tau \geq 1$, and b_t strictly decreases from τ on if $\tau \leq T-2$.

Proof of the claim. Let $\tau = \max\{t \mid g_t > 0\}$ for the optimal consumption pattern (59). This set is not empty as $g_0 > 0$. If $\tau \geq 1$ then it must be that $b_{\tau-1} \geq b^*$ otherwise an increase in $b_{\tau-1}$ would be feasible and strictly increase the objective. Since the utility of F increases in $b_t(1-\beta r(1-b_t))$ for all $t \in \{0;...;T-1\}$ and $r(1-b^*) < 1$, it must be that $b_s = b^*$ for all $s \leq \tau-1$ as this is feasible and dominates any other pattern up to date τ. Finally, $r(1-b^*) < 1$ also implies that b_t must be smaller than b^* from τ on if $\tau \leq T-1$, and strictly decreasing from τ on if $\tau \leq T-2$. This establishes the result.

This optimal consumption pattern implies that for $t \geq 1$ $P_t = P_t^M + \mathbb{1}_{\{t>\tau\}}\alpha_M$. For all $t \in \{0;...;\tau-1\}$, $g_{t+1} > 0$ and so as in the two-dates case, the Sargent-Wallace debt level would come at the finite cost from overborrowing and the arbitrarily small benefit from inflating away R_tX_t. Thus F sticks to the price-level taking debt level. Conversely, for $t \geq \tau$, $g_{t+1} = 0$, and so the Sargent-Wallace issuance is strictly dominant because it does not affect consumption from date $t+1$ on and raises current consumption because it strictly increases the date-$t+1$ resources against which F borrows and $r(1-b)b$ is strictly increasing.

B.4 Proof of Proposition 7

Consider a sequence $(\bar{x}_t, \bar{b}_t)_{t\in\mathbb{N}}$ that satisfies the conditions in Proposition 7. Letting

$$P_t^F \equiv \frac{B_{t-1} - \frac{b_{t-1}^M P_{t-1}}{Q_{t-1}} + R_{t-1}X_{t-1}}{\bar{x}_t + \bar{b}_t}, \tag{63}$$

we define

$$P_t^* = \begin{cases} \max\left\{P_t^M; \frac{R_{t-1}X_{t-1}}{\bar{x}_t}; P_t^F\right\} & \textit{if } P_t^F \leq \max\left\{P_t^M; \frac{R_{t-1}X_{t-1}}{\bar{x}_t}\right\} + \alpha_M, \\ \max\left\{P_t^M; \frac{R_{t-1}X_{t-1}}{\bar{x}_t}\right\} & \textit{otherwise}. \end{cases} \tag{64}$$

Step 1. Fiscal-dominance equilibrium. The strategy profiles associated with this equilibrium are as follows. We go backwards through the stages of a generic date t.

Stage 3: Default and consumption. M transfers any residual income to F. F pays its debt back if possible and consumes any residual income. If this is not possible then F fully defaults and consumes.

Stage 2: Bond market.

- If they expect default at $t+1$ given history and strategy profiles, savers shun bonds, otherwise their investment in bonds $b_t(h_t, R_t, X_t, x_t, B_t, b_t^M)$ and the bond price Q_t are the solutions of the system

$$Q_t B_t = P_t(b_t + b_t^M), \tag{65}$$

$$P_t = Q_t P_{t+1}^* r(1 - b_t - x_t) \tag{66}$$

- M invests either the smallest $b_t^M(h_t, R_t, X_t, x_t, B_t)$ such that $P_{t+1}^* = \max\{P_t^M; R_t X_t / \bar{x}_{t+1}\}$ or $b_t^M = (X_t - R_{t-1}X_{t-1})/P_t$ if this set is empty.

- F issues $B_t(h_t, R_t, X_t, x_t) = B_t^*$ bonds, where

$$B_t^* = (P_{t+1}^M + \alpha_M)\left(\bar{x}_{t+1} + \bar{b}_{t+1} - r(1 - \bar{x}_t - \bar{b}_t)\frac{R_{t-1}X_{t-1}}{P_t}\right). \tag{67}$$

Stage 1: Market for reserves.

- The price P_t and savers' investment in reserves $x_t(h_t, R_t, X_t)$ are the solutions of the system

$$X_t = P_t x_t, \tag{68}$$

$$P_t R_t = P_{t+1}^* r(1 - b_t - x_t), \tag{69}$$

  where all the parameters other than X_t, R_t—that is, b_t and P_{t+1}^*—are given by the strategy profiles above.

- M selects $X_t = R_{t-1}X_{t-1}$, and announces $R_t = r(1 - x_t - b_t)P_{t+1}^*/P_t^*$, where all the future parameters defining R_t are generated by the above profiles.

These strategy profiles have two salient features. First, (63) encodes that the private sector anticipates that the maximum future resources collected in the date-$t+1$ respective reserve and bond markets are $\bar{x}_{t+1}$ and $\bar{b}_{t+1}$, respectively. This pins down the maximum size of the bubble that the public sector can blow. Second, it is weakly dominant for F to issue at the Sargent-Wallace level by issuing B_t^* given by (72) if the bubbles on reserves and bonds are sufficiently small that it is willing to borrow more at a higher rate.

Step 2. Monetary-dominance equilibrium. The strategy profiles and price functions associated with this equilibrium are as follows. We go again backwards through the stages of a generic date t.

Stage 3: Default and consumption. M transfers any residual income to F. F pays its debt back if possible and consumes any residual income. If this is not possible then F fully defaults and consumes.

Stage 2: Bond market.

- If they expect default at $t+1$ given history and strategy profiles, savers shun bonds, otherwise their investment in bonds $b_t(h_t, R_t, X_t, x_t, B_t, b_t^M)$ and the bond price Q_t are the solutions of the system

$$Q_t B_t = P_t(b_t + b_t^M), \tag{70}$$

$$P_t = Q_t P_{t+1}^* r(1 - b_t - x_t) \tag{71}$$

- M invests either the smallest $b_t^M(h_t, R_t, X_t, x_t, B_t)$ such that $P_{t+1}^* = \max\{P_t^M; R_t X_t / \bar{x}_{t+1}\}$ or $b_t^M = (X_t - R_{t-1}X_{t-1})/P_t$ if this set is empty.

- If $x_t > 0$, F issues $B_t(h_t, R_t, X_t, x_t)$ bonds, where

$$B_t(h_t, R_t, X_t, x_t) = P_{t+1}^M \left(\bar{x}_{t+1} + \bar{b}_{t+1}\right) - R_t X_t. \tag{72}$$

  Otherwise, $B_t = 0$.

Stage 1: Market for reserves.

- If either $t = 0$ or $P_{t-1} = P_{t-1}^M$, the price P_t and savers' investment in reserves

$x_t(h_t, R_t, X_t)$ solve the system

$$X_t = P_t x_t, \tag{73}$$

$$P_t R_t = P^*_{t+1} r(1 - b_t - x_t), \tag{74}$$

where all the parameters other than X_t, R_t—that is, b_t and P^*_{t+1}—are given by the strategy profiles above.

Otherwise, $x_t = 0$ *and so* $P_t = +\infty$.

- M selects $X_t = R_{t-1} X_{t-1}$, and announces $R_t = r(1 - x_t - b_t) P^*_{t+1}/P^*_t$, where all the future parameters defining R_t are generated by the above profiles.

There are two differences with the fiscal-dominance equilibrium. First, savers' behavior in the reserve market is now history dependent. If the last price was on target, then they behave in the forward-looking fashion of the fiscal-dominance equilibrium. Otherwise, they shun the reserve market thereby prohibiting the public sector from issuing nominal promises forever. This move is in italics in the description of the strategy profiles. This induces F to adopt the price-level taking debt level. The reason is that M would prefer to force default rather than entering into this autarky economy.

C Formal equilibrium definition

In this section, we define our equilibrium concept adapting the exact same formalism as that in the original definition of Ljungqvist and Sargent (2018) to our context.

As in the core of the text, we recursively define an history $h_{t+1} = \{h_t, R_t, X_t, x_t, B_t, b^M_t, b_t, l_t, g_t\}$. For all dates t, we consider the collection of functions

$$\sigma \equiv (\sigma^M, \sigma^x, \sigma^F, \sigma^m, \sigma^b, \sigma^f) = \{\sigma^M_t, \sigma^x_t, \sigma^F_t \sigma^m_t, \sigma^b_t, \sigma^f_t\}_{t \geq 0}$$

such that M takes decisions $(R_t, X_t) = \sigma^M_t(h_t)$ after observing history h_t, the aggregate investment in reserves of the private sector is $x_t = \sigma^x_t(h_t, R_t, X_t)$, the government bond issuance satisfies $B_t = \sigma^F_t(h_t, R_t, X_t, x_t)$, M's purchases of bonds and dividend policy is $b^M_t = \sigma^m_t(h_t, R_t, X_t, x_t, B_t)$, the private sector invests $b_t = \sigma^b_t(h_t, R_t, X_t, x_t, B_t, b^M_t)$ in bonds and finally the government decides to consume and to repay as follows

$$(g_t, l_t) = \sigma_t^f(h_t, R_t, X_t, x_t, B_t, b_t^M, b_t).$$

Optimality of (R_t, X_t). Given a strategy profile σ and history h_t, the decision $(R_t, X_t) = \sigma_t^M(h_t)$ is optimal when (R_t, X_t) is solution to:

$$U_t^M(\sigma(h_t)) \equiv \max_{R'_t, X'_t} - \mid X'_t/x_t - P_t^M \mid - \alpha_M l_t + \beta U_{t+1}^M(\sigma(h_{t+1}))$$

such that $x_t = \sigma_t^x(h_t, R'_t, X'_t)$, $B'_t = \sigma_t^F(h_t, R'_t, X'_t, x_t)$, $b_t^M = \sigma_t^m(h_t, R'_t, X'_t, x_t, B_t)$, $b_t = \sigma_t^b(h_t, R'_t, X'_t, x_t, B_t, b_t^M)$ and $(g_t, l_t) = \sigma_t^f(h_t, R'_t, X'_t, x_t, B_t, b_t^M, b_t)$.
Finally, $h_{t+1} = \{h_t, R'_t, X'_t, x_t, B_t, b_t^M, b_t, l_t, g_t\}$.

x_t is a competitive outcome. Given a strategy profile σ and the history $\{h_t, R_t, X_t\}$, the aggregate saving decision in reserves $x_t = \sigma_t^x(h_t, R_t, X_t)$ is optimal when x_t is such that:

(i) $P_t = X_t/x_t$, $Q_t = P_t(b_t + b_t^M)/B_t$ and $P_{t+1} = X_{t+1}/x_{t+1}$ where

$$\begin{aligned}
&B_t = \sigma_t^F(h_t, R_t, X_t, x_t),\\
&b_t^M = \sigma_t^m(h_t, R_t, X_t, x_t, B_t),\\
&b_t = \sigma_t^b(h_t, R_t, X_t, x_t, B_t, b_t^M),\\
&(g_t, l_t) = \sigma_t^f(h_t, R_t, X_t, x_t, B_t, b_t^M, b_t),\\
&h_{t+1} = \{h_t, R_t, X_t, x_t, B_t, b_t^M, b_t, l_t, g_t\},\\
&(X_{t+1}, R_{t+1}) = \sigma_{t+1}(h_{t+1}),\\
&x_{t+1} = \sigma_{t+1}(h_{t+1}, R_t, X_t).
\end{aligned}$$

(ii) $(x_t, b_t) \in \arg\max_{(x,b)\in[0,1]^2, x+b\leq 1} \left\{ \left(\frac{R_t P_t}{P_{t+1}} - r(1-x-b) \right) x + \left(\frac{(1-l_{t+1})P_t}{Q_t P_{t+1}} - r(1-x-b) \right) b \right\}.$

Optimality of B_t. Given a strategy profile σ and the history $\{h_t, R_t, X_t, x_t\}$, the decision $B_t = \sigma_t^F(h_t, R_t, X_t, x_t)$ is optimal when B_t solves the following problem:

$$U_t^F(\sigma(h_t, R_t, X_t, x_t)) \equiv \max_{B'_t}(g_t - \alpha_F l_t) + \beta U_{t+1}^F(\sigma(h_{t+1}, R_{t+1}, X_{t+1}, x_{t+1})),$$

such that

$$
\begin{aligned}
&b_t^M = \sigma_t^m(h_t, R_t, X_t, x_t, B_t'),\\
&b_t = \sigma_t^b(h_t, R_t, X_t, x_t, B_t', b_t^M),\\
&(g_t, l_t) = \sigma_t^f(h_t, R_t, X_t, x_t, B_t', b_t^M, b_t),\\
&h_{t+1} = \{h_t, R_t, X_t, x_t, B_t', b_t^M, b_t, l_t, g_t\},\\
&(R_{t+1}, X_{t+1}) = \sigma_{t+1}^M(h_{t+1}),\\
&x_{t+1} = \sigma_{t+1}^x(h_{t+1}, R_{t+1}, X_{t+1}).
\end{aligned}
$$

b_t^M **is optimal.** Given a strategy profile σ and history $\{h_t, R_t, X_t, x_t, B_t\}$, the decision $(b_t^M, \theta_t) = \sigma_t^m(h_t, R_t, X_t, x_t, B_t)$ is optimal when b_t^M is solution to:

$$
U_t^M(\sigma(h_t, R_t, X_t, x_t, B_t)) \equiv \max_{b_t^{',M}} - \mid X_t/x_t - P_t^M \mid -\alpha_M l_t + \beta U_{t+1}^M(\sigma(h_{t+1}, R_{t+1}, X_{t+1}, x_{t+1}, B_{t+1})))
$$

such that $b_t^{',M} \leq x_t(1 - R_{t-1}X_{t-1}/X_t)$ and

$$
\begin{aligned}
&b_t = \sigma_t^b(h_t, R_t, X_t, x_t, B_t, b_t^{',M})\\
&(g_t, l_t) = \sigma_t^f(h_t, R_t, X_t, x_t, B_t, b_t^{',M}, b_t)\\
&hh_{t+1} = \{h_t, R_t, X_t, x_t, B_t, b_t^{',M}, b_t, l_t, g_t\}\\
&(R_{t+1}, X_{t+1}) = \sigma_{t+1}^M(h_{t+1}),\\
&x_{t+1} = \sigma_{t+1}^x(h_{t+1}, R_{t+1}, X_{t+1}),\\
&B_{t+1} = \sigma_{t+1}^F(h_{t+1}, R_{t+1}, X_{t+1}, x_{t+1}).
\end{aligned}
$$

b_t **is a competitive outcome.** Given a strategy profile and the history $\{h_t, R_t, X_t, x_t, B_t, b_t^M\}$, the aggregate saving decision in bonds $b_t = \sigma_t^b(h_t, R_t, X_t, x_t, B_t, b_t^M)$ is a competitive outcome when:

(i) Prices and default decisions are as follows: $P_t = X_t/x_t$, $Q_t/P_t = (b_t + b_t^M)/B_t$,

$P_{t+1} = X_{t+1}/x_{t+1}$, l_{t+1} is given by

$$
\begin{aligned}
&(g_t, l_t) = \sigma_t^f(h_t, R_t, X_t, x_t, B_t, b_t^M, b_t) \\
&h_{t+1} = \{h_t, R_t, X_t, x_t, B_t, b_t^M, b_t, l_t, g_t\} \\
&(X_{t+1}, R_{t+1}) = \sigma_{t+1}^M(h_{t+1}) \\
&x_{t+1} = \sigma_{t+1}^x(h_{t+1}, R_{t+1}, X_{t+1}) \\
&B_{t+1} = \sigma_{t+1}^F(h_{t+1}, R_{t+1}, X_{t+1}, x_{t+1}) \\
&b_{t+1}^M = \sigma_{t+1}^m(h_{t+1}, R_{t+1}, X_{t+1}, x_{t+1}, B_{t+1}) \\
&b_{t+1} = \sigma_{t+1}^b(h_{t+1}, R_{t+1}, X_{t+1}, x_{t+1}, B_{t+1}, b_{t+1}^M) \\
&(g_{t+1}, l_{t+1}) = \sigma_{t+1}^f(h_{t+1}, R_{t+1}, X_{t+1}, x_{t+1}, B_{t+1}, b_{t+1}^M, b_{t+1})
\end{aligned}
$$

(ii) $b_t = \arg\max_{b \leq 1-x_t} \left(\frac{(1-l_{s+1})P_s}{Q_s P_{s+1}} - r(1-b-x_t) \right) b.$

(l_t, g_t) **is optimal.** Given a strategy profile σ and history $\{h_t, R_t, X_t, x_t, B_t, b_t, b_t^M\}$, the decision $(l_t, g_t) = \sigma_t^f(h_t, R_t, X_t, x_t, B_t, , b_t, b_t^M)$ is optimal when (l_t, g_t) is solution to:

$$
\begin{aligned}
&U_t^F(\sigma(h_t, R_t, X_t, x_t, B_t, b_t, b_t^M)) \equiv \max_{B_t'}(g_t - \alpha_F l_t) + \cdots \\
&\cdots \beta U_{t+1}^F(\sigma(h_{t+1}, R_{t+1}, X_{t+1}, x_{t+1}, B_{t+1}, b_{t+1}, b_{t+1}^M)), \\
&\text{s.t.} g_t + l_t \frac{B_{t-1}}{P_t} \leq \frac{Q_t B_t}{P_t} + \theta_t
\end{aligned}
$$

where $P_t = X_t/x_t$, $Q_t = P_t(b_t + b_t^M)/B_t$, $\theta_t = l_t b_{t-1}^M P_{t-1}/(Q_{t-1}P_t) + x_t(1 - R_{t-1}X_{t-1}/X_t) - b_t^M$ and

$$
\begin{aligned}
&h_{t+1} = \{h_t, R_t, X_t, x_t, B_t, b_t^m, b_t, l_t', g_t'\} \\
&(R_{t+1}, X_{t+1}) = \sigma_{t+1}^M(h_{t+1}), \\
&x_{t+1} = \sigma_{t+1}^x(h_{t+1}, R_{t+1}, X_{t+1}), \\
&B_{t+1} = \sigma_{t+1}^F(h_{t+1}, R_{t+1}, X_{t+1}, x_{t+1}), \\
&b_{t+1}^M = \sigma_{t+1}^m(h_{t+1}, R_{t+1}, X_{t+1}, x_{t+1}, B_{t+1}), \\
&b_{t+1} = \sigma_{t+1}^b(h_{t+1}, R_{t+1}, X_{t+1}, x_{t+1}, B_{t+1}, b_{t+1}^M).
\end{aligned}
$$

These definitions formalize the requirement in the equilibrium definition in the body of the paper that all agents hold the belief that future actions are taken according to the strategy profile σ.

Equilibrium definition. An equilibrium is a strategy profile $\sigma = (\sigma^M, \sigma^x, \sigma^F, \sigma^m, \sigma^b, \sigma^f)$ such that, for any period t and history h_t, we have:

(i) Given h_t and σ, $(R_t, X_t) = \sigma_t^M(h_t)$ is optimal.

(ii) Given $\{h_t, R_t, X_t\}$ and σ, $x_t = \sigma_t^x(h_t, R_t, X_t)$ is a competitive outcome.

(iii) Given $\{h_t, R_t, X_t, x_t\}$ and σ, $B_t = \sigma_t^F(h_t, R_t, X_t, x_t)$ is optimal.

(iv) Given $\{h_t, R_t, X_t, x_t, B_t\}$ and σ, $b_t^M = \sigma_t^m(h_t, R_t, X_t, x_t, B_t)$ is optimal.

(v) Given $\{h_t, R_t, X_t, x_t, B_t, b_t^M\}$ and σ, $b_t = \sigma_t^b(h_t, R_t, X_t, x_t, B_t, b_t^M)$ is a competitive outcome.

(vi) Given $\{h_t, R_t, X_t, x_t, B_t, b_t, b_t^M\}$ and σ, $(l_t, g_t) = \sigma_t^f(h_t, R_t, X_t, x_t, B_t, b_t, b_t^M)$ is optimal.

D $T \geq 2$ and $r(1-b^*) \geq 1$

This appendix describes the equilibrium in the case in which $T \geq 2$ and $r(1-b^*) \geq 1$. The following ingredients are useful to describe the equilibria. Let $\phi(b) = br(1-b)$ for $b \in [0,1)$. For $t \geq 1$, let

$$b_t^* = \arg\max_{b \in [0,1)} \{b - \beta^t \phi^{(t)}(b)\}, \tag{75}$$

where the notation $\phi^{(t)}$ corresponds to the function ϕ composed t times. Notice that b_1^* corresponds to b^* in the body of the paper, where we dropped the subscript 1 for notational parsimony. For brevity we restrict the analysis to the case in which $b_1^* > 0$.

Proposition 8. ***(Endogenous regime switching when*** $r(1-b_1^*) \geq 1$***)*** *Suppose* $r(1-b_1^*) \geq 1$. *There exists a unique equilibrium.* M *does not issue new reserves between dates 0 and* $T-1$. *At date 0,* F *has strictly positive consumption* ($g_0 > 0$) *and* $P_0 = P_0^M$. *There is no consumption at the interim dates:* $g_t = 0$ *for* $t \in \{1; ...; T-1\}$.

1. *If* $\bar{x} + \bar{b} > \phi^{(T)}(b_T^*)$, $g_T > 0$ *and there is monetary dominance at every date.*

2. If $\bar{x}+\bar{b} \leq \phi(b_1^*)$, then $g_T = 0$ and there is fiscal dominance at every date $t \geq 1$.

3. In the interim range $\bar{x}+\bar{b} \in (\phi(b_1^*), \phi^{(T)}(b_T^*)]$, then $g_T > 0$ is arbitrarily small, and there exists $\tau \in \{1;...;T-1\}$ such that there is fiscal dominance until τ and monetary dominance afterwards.

Proof. See Appendix D.1. □

Case 1. is the outright extension to T periods of the situation in which $\hat{g}_1(0) > 0$ when $T = 1$. In this case, the optimal consumption pattern of F implies $g_T > 0$ and the Sargent-Wallace debt level would distort it at excessively small gains if $R_{-1}X_{-1}$ is sufficiently small. F finds the Sargent-Wallace debt level unpalatable both at date 0 and subsequently: It rolls over a debt burden that is ex-post excessive and that it is not willing to further increase ($b_t > b_{T-t}^*$ for $t \in \{1;...;T-1\}$).

Case 2. is the outright extension of the situation in which $\hat{g}_1(0) = 0$ when $T = 1$. In this case, F is constrained by its next-date resources at each date and enjoys the extra slack generated by the Sargent-Wallace debt level, which M and savers anticipate along the equilibrium path.

Case 3. is more complex. Even though optimal consumption would require $g_T = 0$ since $\bar{x}+\bar{b} \leq \phi^{(T)}(b_T^*)$, F leaves a little bit of terminal resources on the table. It leaves just enough that it is not tempted by the Sargent-Wallace debt level at τ, the date at which the value of debt rolled over since date 0 snowballs above the ex-post optimal level $b_{T-\tau}^*$. Suppose by contradiction that the equilibrium is such that $g_T = 0$, and that F borrows at the Sargent-Wallace level at all dates. Then M would optimally deviate and force default by setting P_τ at P_τ^M instead of the value $P_\tau^M + \alpha_M$ along the equilibrium path. F would then prefer to raise debt at the optimal level (arbitrarily close to $b_{T-\tau}^*$ given an arbitrarily small $R_{-1}X_{-1}$) and to allow for monetary dominance at $\tau+1$. With this deviation, M incurs the same date-τ disutility α_M as along the equilibrium path but gains future price levels on target. Thus $g_T = 0$ cannot be an equilibrium and F must leave (an arbitrarily small amount of) money on the table at date 0.

An interesting feature of equilibrium in this latter case 3. is that fiscal dominance prevails until τ even though $g_T > 0$. At face value, this contradicts the two-dates insight that fiscal dominance requires that F pledges its entire future tax capacity. With more than two dates, the reason F may credibly be unable to reduce consumption in the future is that, as we have just seen, this would make the Sargent-Wallace debt level dominant, and in turn lead M to force default. But then savers anticipating such future default would not lend and default would occur right away.

D.1 Proof of Proposition 8

Claim 1. We show that if $r(1-b_1^*) > 1$, the sequence $(b_t^*)_{t\geq 1}$ is such that $\phi(b_{t+1}^*) > b_t^* > b_{t+1}^*$.

Proof of the claim. The proof is by recursion. The first-order condition implicitly defining b_2^* is

$$\beta^2(\phi^{(2)})'(b) = 1 \tag{76}$$

or

$$\beta^2\phi'(\phi(b))\phi'(b) = 1. \tag{77}$$

If $b_2^* > b_1^*$ then $r(1-b_2^*) > 1$ and $\phi(b_2^*) > b_2^* > b_1^*$ in which case (77) and thus (76) cannot hold because ϕ is convex increasing. Thus it must be that $b_2^* < b_1^*$ in which case (77) implies $\phi(b_2^*) > b_1^*$. For $t \geq 2$, the first-order condition implicitly defining b_{t+1}^* is

$$\beta^{t+1}(\phi^{(t+1)})'(b) = 1 \tag{78}$$

or

$$\beta^{t+1}(\phi^{(t)})'(\phi(b))\phi'(b) = 1. \tag{79}$$

or

$$\beta^{t+1}\phi'(\phi^{(t)}(b))(\phi^{(t)})'(b) = 1. \tag{80}$$

It must be that $\phi(b_{t+1}^*) > b_t^*$. Otherwise, (79) implies $b_{t+1}^* \geq b_1^*$ and so $\phi(b_{t+1}^*) > b_{t+1}^* \geq b_1^* > b_t^*$, a contradiction. Applying this to previous dates yields from the recursion hypothesis $\phi^{(t)}(b_{t+1}^*) > b_1^*$. But then (80) implies $b_{t+1}^* < b_t^*$.

Claim 2. If $r(1-b_1^*) > 1$, the solution to program (59) is such that $g_t = 0$ for all $t \in \{1; ...; T-1\}$.

Proof of the claim. Suppose that the optimal consumption pattern is such that for some $t \in \{0; ...; T-2\}$, $r(1-b_t)b_t < b_{t+1}$. It must then be that $b_t \geq b^*$ otherwise an increase in b_t would strictly increase the objective and be feasible. This implies that $b_{t+1} > b_t \geq b^*$, a contradiction, as decreasing b_{t+1} would strictly increase the objective.

We now prove the proposition, studying in turn each of the three relevant ranges of value.

1. $\bar{x} + \bar{b} > \phi^{(T)}(b_T^*)$**.** In this case F is not constrained by its terminal resources when choosing initial borrowing and so finds the Sargent-Wallace debt level at date 0 unpalatable. Claim 1 implies $\phi(b_{t+1}^*) > b_t^*$ for $t \in \{1; ...; T-1\}$. Since b_0 is arbitrarily close to b_0^* for $R_{-1}X_{-1}$ sufficiently small, $b_1 = \phi(b_0) > b_{T-1}^*$, and Claim 1 and that ϕ is increasing then implies that $b_t > b_{T-t}^*$ for all $t \in \{1; ...; T-1\}$. This implies that F finds the Sargent-Wallace debt level costly at all future dates because it already borrows more than if it had no legacy debt under the price-level taking debt level.

2. $\bar{x}+\bar{b} \leq \phi(b_1^*)$**.** In this case F always issues the Sargent-Wallace debt level because it is constrained by its future resources at every date. Formally, $b_{T-t} < b_t^*$ for all $t \in \{1; ...; T\}$. It is true for $t = 1$ since $b_{T-1} = \phi^{-1}(\bar{x} + \bar{b}) \leq b_1^*$, and then by recursion since Claim 1 implies

$$b_{T-t} = \phi_{-1}(b_{T-t+1}) < \phi_{-1}(b_{t-1}^*) < b_t^*. \tag{81}$$

3. $\phi(b_1^*) < \bar{x} + \bar{b} \leq \phi^{(T)}(b_T^*)$**.** In this case F is constrained by its terminal resources since these do not exceed $\phi^{(T)}(b_T^*)$. The optimal consumption pattern thus dictates that F borrow against its entire terminal resources—$g_T = 0$—and rolls over its debt. We show that this cannot be the equilibrium consumption pattern. If this were the case, then there would be fiscal dominance from date 1 on since the Sargent-Wallace debt level increases the current consumption of F while leaving future ones unchanged at zero. From Claim 1, there exists $\tau \in \{1; ...; T-1\}$ such that $b_t \leq b_{T-t}^*$ for $t < \tau$ and $b_t > b_t^*$ afterwards. From τ on, M is better off setting the price level on target and let the government default at τ as F would use this slack to reduce its borrowing rather than forcing fiscal dominance since it is strictly above its ex-post maximum borrowing b_t^*. So F cannot borrow so much at date 0 that it strictly prefers to force fiscal dominance after τ, as savers anticipating future default would not lend. Before τ however, F is still constrained by its future resources and issues the Sargent-Wallace debt level.