# The Central Bank, the Treasury, or the Market: Which One Determines the Price Level?*

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**Abstract**

This paper studies a model in which the price level is the outcome of dynamic strategic interactions between a fiscal authority, a monetary authority, and investors in government bonds and reserves. The "unpleasant monetarist arithmetic", whereby aggressive fiscal expansion forces the monetary authority to chicken out and to lose control of inflation, occurs only if the public sector lacks fiscal space, in the sense that public debt along the optimal fiscal path gets sufficiently close to the threshold above which the fiscal authority would find default optimal. Otherwise, monetary dominance prevails even though the central bank has neither commitment power nor fiscal backing.

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# 1 Introduction

Public sectors in most major economies have issued since 2008 an amount of liabilities, both government debt and central-bank reserves, that is unprecedented in peacetime. Their resulting fiscal positions have led a number of observers to worry about the ability of their independent central banks to fulfill the price-stability part of their mandates going forward. Vindicating these concerns, the realizations of inflation have recently, and for the first time in decades, been significantly above target in both the US and the eurozone.

The theoretical underpinning of these concerns can be traced back to Sargent and Wallace's "unpleasant monetarist arithmetic" (Sargent and Wallace, 1981). This paper shows that if a fiscal authority embarks on a path of aggressive debt issuance and deficits, the monetary authority has no option but generating sufficient seigniorage income despite the inflationary consequences. This seminal work has initiated a large body of research studying the respective contributions of fiscal and monetary policies to the determination of the price level.

In Sargent and Wallace (1981), the strategic interactions between the branches of government respectively in charge of fiscal and monetary policies are a simple two-stage sequential game. The fiscal authority "moves first" in the sense that it commits at the outset to a path of debt issuance and deficits for the entire future. As a second mover, the monetary authority, because it cares by assumption about the government's solvency, has no choice but accommodating this path. This particular case is theoretically important because it showcases the pioneering insight that fiscal policy may tie the hands of a formally independent central bank. If it moves first, the fiscal authority can dictate the price level even though it does not conduct monetary policy.

In practice, however, fiscal and monetary authorities simultaneously implement their respective policies, interacting repeatedly and without eternal commitment. It is a priori unclear which authority, if any, imposes its objectives on the other. As Sargent and Wallace (1981) put it in conclusion of their unpleasant arithmetic: *"The question is, Which authority moves first, the monetary authority or the fiscal authority? In other words, Who imposes discipline on whom?"*.

This paper solves a model of dynamic strategic interactions between a fiscal authority, a monetary one, and investors in their respective liabilities. All agents repeatedly interact without commitment, and so which authority imposes its objectives is endogenously driven by the primitives of the economy. Our goal is to identify the circumstances under which the fiscal authority takes the price-level targets set by the monetary authority as given, and that in which by contrast monetary policy accommodates fiscal policy.

We study an economy in which an independent monetary authority seeks to control the price level. It is independent in the sense that it sets its own price-level target, and has a free hand at managing its balance sheet in order to reach it.[1] The monetary authority issues reserves that are the unit of account of the economy—The price of a consumption unit in terms of reserves is the price level. It also decides on the nominal interest rate on reserves, on the investment of the proceeds from issuing reserves, and on possible transfers ("dividends") to the fiscal authority.

The fiscal authority seeks to spend optimally. It issues nominal bonds and uses the proceeds to spend or/and to repay all or part of maturing bonds. It can also raise distortive taxes. Walrasian private investors form optimal portfolio of reserves and government bonds.

We solve for the subgame-perfect Nash equilibria resulting from their interactions with a focus on the resulting price level. We deem "monetary dominance" the situation in which the equilibrium price level corresponds to the target of the monetary authority. "Fiscal dominance" is the alternative in which the price level exceeds this target, and reaches instead a higher level that is consistent with the solvency of the public sector.

The fiscal authority has an ex-post strict preference for inflation as it erodes the value of outstanding public liabilities, thereby allowing for more spending holding taxes fixed. Unlike in Sargent and Wallace (1981) though, the fiscal authority must find a way to commit to the type of fiscal expansion that would induce such an inflationary path. It must credibly establish that if the future price level is too low, it will prefer outright default to making good on its debt by raising taxes or/and cutting expenditures. The only way it can commit to such a future preference for default conditional on low inflation is by frontloading expenditures and financing them with a sufficiently large current debt issuance. This commitment device is costly, however, in comparison with the smoother optimal fiscal path that takes price levels as given. If such credible fiscal expansion is too unbalanced relative to the smoother optimal fiscal path, then the fiscal authority does not enter into it. it. In this case there is monetary dominance: The central bank has a

[1]Section 3.6 discusses the case in which the fiscal authority can renege on such independence.

free hand at implementing its price-level target despite its own inability to commit.

In sum, the fiscal authority *can* always force the monetary one to inflate away legacy public liabilities by issuing enough public debt. However, it *wants* to do so only if the benefits of this inflationary fiscal expansion more than offset its costs. The benefits depend on the size of the legacy liabilities that are to be inflated away, and on the maximum amount of inflation that the central bank is willing to tolerate to avert sovereign default. The central bank can partially control the size of legacy liabilities by maintaining the lowest possible volume of outstanding reserves.

The costs to the fiscal authority from embarking on an unbalanced inflationary fiscal path have three main drivers. First, if the fiscal authority incurs very large costs of outright default (e.g., market exclusion), then the inflationary path must be very unbalanced because only a very large future tax burden will lead the fiscal authority to credibly contemplate default if inflation is too low. Second, large costs of taxation also make the inflationary path unpalatable, as this path typically involves larger future taxes than the optimal one. Third, since the inflating path involves more debt than the optimal one taking price levels as given, the interest rate that the market requires at such debt levels makes this path unpalatable if it is sufficiently large relative to the fiscal authority's discount rate.

In sum, monetary dominance prevails if the public sector has sufficient fiscal space, in the sense that at any point along the optimal fiscal path taking price levels as given, the fiscal authority would prefer to respond to an exogenous increase in public liabilities with an increase in taxes or/and a reduction in expenditures rather than with formal default. Conversely, if the optimal fiscal path gets sufficiently close to the default boundary, then the fiscal authority may deviate from it, and double down on debt in order to force the monetary authority to erode public liabilities through inflation. Overall, we provide a strategic framework that confirms and rationalizes the common intuition according to which fiscal dominance is a concern when the public sector is distressed, whereas a formally independent central bank can conduct monetary policy without fiscal interference in normal times.

We first present our main insights in the simplest possible model with two dates. In this model, fiscal and monetary authorities incur exogenous costs in case of sovereign default. We then study an infinite-horizon, dynamically inefficient economy in which public liabilities are Ponzi schemes. We endogenize default costs in this economy. They result from investors in bond and reserve markets downsizing the size of the Ponzi schemes that they believe—in a self-justified fashion—to be sustainable in case default occurs.[2] In this case, the extent to which investors run not only on debt but also on reserves in case of sovereign default drives the monetary authority's willingness to accommodate fiscal expansion. The central bank is all the more willing to avoid default with some current inflation because it faces the risk of hyperinflation following formal default.

**Related literature.** Our paper belongs to the very rich literature on optimal fiscal and monetary policies following Calvo (1978) and Lucas and Stokey (1983). As envisioned in this literature, nominal public liabilities lead to a time-inconsistency problem for public authorities. Furthermore, this literature has also discussed the importance for this time-inconsistency problem of the public sector's net nominal liabilities, i.e., nominal debt and money in the hands of the private sector (see Alvarez et al., 2004; Persson et al., 2006, among others). In our framework, delegation of monetary tools to the monetary authority helps solve the time-inconsistency of the government, but imperfect delegation due to limited commitment creates a game between fiscal and monetary authorities.

From this perspective, we are connected to the literature on the interactions between monetary and fiscal policies pioneered by Sargent and Wallace (1981) (see Leeper, 1991; Sims, 1994; Woodford, 1994, 1995; Cochrane, 2001, 2005; McCallum, 2001; Buiter, 2002; Niepelt, 2004; Jacobson et al., 2019, among others). As in Sargent and Wallace (1981), the monetary authority can adjust seigniorage revenue to help the fiscal authority satisfy its budget constraint. The simple economy in which we cast our game of chicken relates in particular to that in which Bassetto and Sargent (2020) study fiscal and monetary interactions. Our paper is also closely connected to the papers that identify fiscal requirements such that the central bank can attain its price stability objective, including fiscal rules (e.g. Woodford, 2001) or a ring-fenced balance sheet (e.g Sims, 2003; Bassetto and Messer, 2013; Hall and Reis, 2015; Benigno, 2020). Closer to our paper, Martin (2015) finds as we do that fiscal irresponsibility leads to long-term inflation. Finally, Coibion et al. (2021) provide causal evidence that private agents do anticipate inflationary effects of fiscal policy: Their evidence that households associate future debt levels with inflation

[2] Whereas the punishment through market exclusion in the pioneering work of Eaton and Gersovitz (1981) is not renegotiation-proof, we only consider subgame-perfect equilibria.

is consistent with our model's result that future net public liability is a key determinant of central bank's future incentives to inflate. In line with this literature, our paper aims at precisely describing the respective markets in which fiscal and monetary authorities intervene, as well as their instruments and budget constraints. Our contribution is to explicitly model the strategic interactions between fiscal and monetary authorities in such an environment.

That fiscal and monetary authorities may have ex-post conflicting objectives is a natural assumption. This has been in fact the main rationale behind setting up independent central banks. This is also motivated by the large set of evidence that authorities do not necessarily cooperate and, instead, try to impose their views on each other (see, e.g., Mee (2019) for a historical analysis of the rise of an independent Bundesbank, Silber (2012) for the Volker era, and Bianchi et al. (2019) for evidence that markets reacted to Trump's comments on monetary policy). In this respect, this makes our paper closer to an older literature (Alesina, 1987; Alesina and Tabellini, 1987; Tabellini, 1986, e.g.) that investigates the equilibria of games between multiple branches of government. More recent contributions include Dixit and Lambertini (2003) or the literature that explores disciplining mechanisms for the public sector in models following Barro and Gordon (1983a,b), such as Halac and Yared (2020).

With respect to this literature, our contribution is to provide an explicit set of instruments to both the fiscal and the monetary authorities as well as a game-theoretic foundation to fiscal and monetary interactions. Our approach of the resulting macroeconomic game follows Chari and Kehoe (1990), Stokey (1991) and Ljungqvist and Sargent (2018) but extended to multiple large agents and markets. In particular, our approach to model markets follows Bassetto (2002) as, in our setting, price levels as well as debt prices are market equilibrium objects.

Finally, our paper relates to the recent literature that compares formal sovereign default and default in the form of inflation (Bassetto and Galli, 2019; Galli, 2020). We cover the case in which distinct branches of government control each tool and act non-cooperatively. The infinite-horizon model offers a novel way of endogenizing the respective costs of each type of default.

# 2 Two-Date Model: Setup

Our model features a fiscal authority and a monetary one that interact strategically. They also interact with the private sector in the markets for their respective liabilities. The monetary authority issues reserves that are the unit of account of the economy, and seeks to control the price level. The fiscal authority seeks to spend optimally and issues nominal bonds.

There are two dates indexed by $t \in \{0; 1\}$. There is a single consumption good. We describe in turn the private and public sectors.

**Private sector.** The private sector is comprised of a unit mass of agents, deemed "savers", who are each endowed with a large quantity of the consumption good at dates 0 and 1. They rank consumption streams $(c_0, c_1)$ according to the criterion

$$c_0 + \frac{c_1}{r}, \tag{1}$$

where $r > 0$.

**Public sector.** The public sector features a fiscal authority $F$ and a monetary authority $M$.

**Monetary authority.** The monetary authority issues reserves and announces the interest rate $R$ on them. Reserves trade for the consumption good in date-0 and date-1 markets for reserves. Reserves are the unit of account of the economy. We denote by $P_t$ the price level—the price of the consumption good in terms of reserves in the date-$t$ market for reserves. Let also $X_t$ denote the quantity of outstanding reserves at the end of date $t$, and $x$ denote the endogenous quantity of goods that savers bid for reserves in the date-0 market for reserves. As detailed below, the terminal date-1 demand for reserves will be an exogenous quantity $\bar{x}$ in this two-date model.[3] We also assume that some legacy reserves $R_{-1}X_{-1} \geq 0$ are sold in the date-0 reserve market by some unmodelled agents—for example, by savers born at date -1 and seeking to consume at date 0.

$M$ can also transfer resources to $F$ ("pay a dividend"), and $\theta_t$ denotes the real date-$t$ transfer from $M$ to $F$.

[3] This demand $\bar{x}$ will be endogenous in the infinite-horizon version of the model in Section 6.

**Fiscal authority.** The fiscal authority issues one-period nominal bonds at date 0. A bond is a claim to one unit of account at date 1. Both savers and $M$ can trade goods for bonds. Let $B$ denote the number of bonds issued by $F$ at date 0, $Q$ the price at which they are sold (in terms of reserves), and $b$ and $b^M$ the respective quantities of goods that savers and $M$ respectively trade for bonds in the bond market.

The fiscal authority can tax savers' date-1 endowment. Collecting taxes $\tau \geq 0$ comes at a utility cost $c(\tau)$ to $F$ (as shown below in its preferences (6)).[4] $F$ also consumes. Let $g_t$ denote its date-$t$ consumption.

Finally, $F$ decides at date 1 on the haircut or loss given default $l \in [0, 1]$ that it applies to its maturing bonds. A haircut $l$ means that bondholders receive $(1-l)$ units of account per bond.

## 2.1 Extensive-form game

The detail of the timing according to which the agents take the above actions is as follows.[5] The game is one of public information, and so each action is conditional on the entire history, which we omit in the notations for simplicity.

**Date-0 market for reserves.**

1. $M$ selects total date-0 outstanding reserves $X_0 \geq R_{-1}X_{-1}$ by issuing new reserves $X_0 - R_{-1}X_{-1}$ on top of $R_{-1}X_{-1}$ sold by old savers, and announces the interest rate $R \geq 0$ between dates 0 and 1 on them.[6]

2. Savers invest an aggregate quantity $x \geq 0$ of consumption units in the market for reserves at the price level $P_0$.

**Date-0 bond market.**

3. $F$ issues $B \geq 0$ bonds.

4. $M$ invests $b^M \in [0, (X_0 - R_{-1}X_{-1})/P_0]$ consumption units in the bond market.

[4] We could also allow for taxation of the date-0 endowment, but this would slightly burden the analysis without generating additional insights.

[5] Section 3.6 discusses alternative timing assumptions.

[6] We could endow $M$ with consumption units at date 0 that it could use to buy back and cancel all or part of the legacy reserves $R_{-1}X_{-1}$ without affecting the analysis. The remaining net legacy reserves would then be the variable of interest.

5. Savers invest $b \geq 0$ aggregate consumption units in the bond market at a bond price $Q$.

**Date-0 spending.**

6. $F$ selects consumption $g_0$ such that

$$\frac{QB}{P_0} + \theta_0 = g_0, \tag{2}$$

where the dividend $\theta_0$ paid by $M$ is

$$\theta_0 = \frac{X_0 - R_{-1}X_{-1}}{P_0} - b^M. \tag{3}$$

**Date-1 reserve market.**

7. $M$ receives an exogenous terminal demand for reserves $\bar{x}$ from unmodelled agents and issues $X_1 - RX_0 \geq 0$ at a price level $P_1$.

**Date-1 default, taxation, and spending.**

8. $F$ raises taxes $\tau$, and decides on $l \in [0, 1]$ and $g_1$ such that

$$g_1 = \tau + \theta_1 - \frac{(1-l)B}{P_1}, \tag{4}$$

where the dividend $\theta_1$ paid by $M$ is equal to

$$\theta_1 = \frac{X_1 - RX_0}{P_1} + \frac{(1-l)b^M P_0}{QP_1}. \tag{5}$$

A strategy profile $\sigma = (R, X_0, x, P_0, B, b^M, b, Q, X_1, P_1, l, \tau)$ describes all the above actions for each agent given all possible histories.[7]

[7] The strategy profile $\sigma$ does not feature the variables $\theta_0$, $g_0$, $\theta_1$, and $g_1$ as they mechanically derive from the others from (2), (3), (4), and (5).

## 2.2 Objectives of $F$ and $M$

The objectives that $F$ and $M$ respectively seek to maximize are respectively:

$$U^F = v(g_0) + \beta\left(v(g_1) - c(\tau) - \alpha_F \delta\right), \tag{6}$$

$$U^M = - \mid P_0 - P_0^M \mid - \beta_M \mid P_1 - P_1^M \mid - \beta_M \alpha_M \delta, \tag{7}$$

where $\delta = \mathbb{1}_{\{l>0\}}$, $\beta, \beta_M \in (0,1)$, $\alpha_F, \alpha_M > 0$, $v$ is an increasing function, and $P_0^M, P_1^M > 0$. In words, the variable $\delta$ is equal to 1 in case of an outright default on a government bond due at date 1, and to 0 otherwise. Thus, each authority $X \in \{F; M\}$ incurs a cost $\alpha_X$ in case of sovereign default. The fiscal authority also values spending and incurs costs of taxation but does not care about the price level, whereas the monetary authority also finds it costly to deviate from a given target $P_t^M$ for the date-$t$ price level.[8] Taxation costs can be interpreted as distortions that the fiscal authority cares about, or more broadly as any political costs. Our results would carry over if we assumed that $M$ and $F$ both cared about price level and government expenditures, albeit with sufficiently different weights. The assumed stark difference in objectives simplifies the exposition.

We assume that holding (7) fixed, $M$ prefers to maximize (6). Such lexicographic preferences only serve to eliminate equilibria that would crucially rely on $M$ not caring at all about the government's consumption.

**Comments on default costs.** In the pionnering paper of Sargent and Wallace (1981), the preferences of the fiscal and monetary authorities are not spelled out. Yet it is implicit and important that the monetary authority has an arbitrarily large aversion to outright sovereign default. It would otherwise not be willing to accommodate, no matter the inflationary consequences, whichever path of debt and deficits the fiscal authority announces. The cost $\alpha_M$ is finite here, and is only one of the parameters that will determine whether fiscal or monetary dominance prevails. The costs of default $\alpha_F$ and $\alpha_M$ are exogenous in this two-date version of the model, savers will create fully endogenous default costs in the infinite-horizon analysis in Section 6 through market exclusion. Costs from formal default include in practice output losses due to financial-market exclusion or/and trade sanctions, legal and settlement costs, banking crises and more generally

[8]Results would be similar with an inflation target.

financial instability, as well as private costs—electoral or more generally political costs for the fiscal authority and career concerns for central bankers.

### 2.3 Equilibrium concept

**Definition 1.** ***(Equilibrium)*** *An equilibrium is a strategy profile $\sigma$ such that:*

1. *Each action by $F$ and $M$ is optimal given history and its beliefs that the future actions are taken according to the strategy profile.*

2. *Saver $i \in [0,1]$ optimally invests $x^i = x$ in the reserve market given $(R, X_0, x, P_0)$, and the strategy profiles for all future actions, and optimally invests $b^i = b$ in the bond market given $(R, X_0, x, P_0, B, b^M, b, Q)$, and the strategy profiles for all future actions.*

3. *The market for reserves clears at date 0, $P_0 x = X_0$, at date 1, $P_1 \bar{x} = X_1$, and the market for bonds clears at date 0, $QB = P_0(b + b^M)$.*

Our equilibrium concept is that of Ljungqvist and Sargent (2018), which adapts plain game-theoretic subgame perfection to the situation in which a "large" player interacts with Walrasian agents. We extend this concept to the case in which there are two such large players, a monetary and a fiscal authority. Very intuitively, $F$ and $M$ play against "the private sector", which responds to their supply of reserves and bonds with aggregate demands and prices in reserve and bond markets. In equilibrium, these "actions" of the private sector correspond to prices and aggregate quantities such that markets clear, and such that the behavior of each (price-taking) individual saver is optimal given prices and fiscal and monetary policies.

## 3 Baseline Model

This section solves for a baseline version of the model that showcases our central insight in the simplest fashion. We suppose:

**Assumption 1.** ***(Baseline model)***

- *There exists* $\bar{\tau} \geq 0$ *such that* $c(\tau) = 0$ *for* $\tau \leq \bar{\tau}$*, and* $c(\tau)$ *is arbitrarily large for* $\tau > \bar{\tau}$*.*
- *There exists* $\underline{g} \geq 0$ *such that* $v(g) = g$ *for* $g \geq \underline{g}$*, and* $v(g)$ *is arbitrarily small for* $g < \underline{g}$*.*
- $\alpha_F \geq \bar{x} + \bar{\tau} - \underline{g}$. (8)
- $\frac{R_{-1}X_{-1}}{P_0^M} \leq \frac{\bar{x}}{r}$ *and* $\bar{\tau} \geq (1+r)\underline{g}$. (9)

As detailed below, Assumption 1 greatly simplifies the date-1 spending, taxation, and default decisions of the fiscal authority because it implies that $F$ chooses to default if and only if making good on its bond requires spending less than $\underline{g}$ or/and taxing more than $\bar{\tau}$. Section 4 studies a more general cost of taxation.

We solve the game backwards. We first characterize how the fiscal authority $F$ decides on taxation, spending, and default at the final stage of date 1, and then how the monetary authority $M$, rationally anticipating this, optimally sets the date-1 price level in the date-1 reserve market. We then move on to date 0, studying date-0 debt issuance by the fiscal authority. This is the keystone of the analysis, showing how date-0 public debt issuance may lead to what we will deem either fiscal or monetary dominance at date 1. Finally, we analyze monetary policy in the initial reserve market to characterize the equilibrium outcome.

### 3.1 Date-1 default

At the terminal stage of date 1, it is dominant for the fiscal authority to raise taxes no smaller than $\bar{\tau}$ as this comes at no cost and generates resources that can be used for debt repayment or/and spending. The expression of $F$'s terminal consumption as a function of all other actions given by (4), together with $X_1 = P_1\bar{x}$, shows that $F$ can avoid default while spending at least $\underline{g}$ and taxing less than $\bar{\tau}$ if and only if:

$$P_1(\bar{x} + \bar{\tau} - \underline{g}) \geq RX_0 + B - \frac{b^M P_0}{Q}. \tag{10}$$

Condition (10) admits a straightforward interpretation. The left-hand term is the nominal value of total public resources net of incompressible expenditures at date 1 and the right-hand term is the net total liabilities of the public sector, that is, the liabilities in the hands of the private sector, equal to the gross liabilities $RX_0 + B$ minus holdings of government debt by the monetary authority $b^M P_0/Q$.

Condition (8) implies that $F$ does not default if (10) holds because the default cost exceeds the resulting additional spending $(B - b^M P_0/Q)/P_1$. If (10) fails to hold, then $F$ defaults, which warrants $g_1 = \bar{\tau} + \bar{x} - RX_0/P_1 > \underline{g}$ because $\bar{\tau} > \underline{g}$ from condition (9) and $\bar{x} = X_1/P_1 \geq RX_0/P_1$.

In sum, $F$ never taxes above $\bar{\tau}$ nor spends below $\underline{g}$, and $F$ defaults if and only if the solvency condition (10) fails to hold.

## 3.2 Date-1 price level

In the date-1 reserve market, the monetary authority $M$ can set any date-1 price level $P_1 \geq RX_0/\bar{x}$, by issuing $X_1 - RX_0 = \bar{x}P_1 - RX_0$ new reserves. In particular it can set $P_1$ sufficiently large that (10) holds and no default occurs. A larger price level $P_1$ frees up resources available for bond repayments by eroding the real value of outstanding reserves $RX_0$, and reduces the real value of maturing bonds $B$. We denote by $P^F$ the smallest price level such that solvency constraint (10) holds:

$$P^F \equiv \frac{RX_0 + B - \frac{b^M P_0}{Q}}{\bar{x} + \bar{\tau} - \underline{g}}. \tag{11}$$

By definition, expenditures are at the incompressible level ($g_1 = \underline{g}$) as soon as $P_1 = P^F$ so that (10) holds with equality.

Let us also define the date-1 price level $\underline{P}_1$ that $M$ would choose in the absence of solvency constraint:

$$\underline{P}_1 \equiv \max\left\{P_1^M; \frac{RX_0}{\bar{x}}\right\}. \tag{12}$$

If $P^F \leq \underline{P}_1$, then $M$ optimally sets $P_1 = \underline{P}_1$ as it minimizes the departure from its target $\mid P_1 - P_1^M \mid$ (possibly to 0 if $\underline{P}_1 = P_1^M$) without triggering default.

If $P^F > \underline{P}_1$, if $M$ lets $F$ default then it incurs a cost $\alpha_M$ and can and optimally does set the date-1 price level at $\underline{P}_1$. If conversely $M$ seeks to avert default, then it optimally does so by setting the date-1 price at $P^F$, thereby reducing $F$'s consumption to the incompressible level $\underline{g}$. As a result, $M$ prevents $F$ from defaulting by setting $P_1 = P^F$ if and only if $P^F \leq \underline{P}_1 + \alpha_M$.

The following proposition summarizes these results.

**Proposition 1.** ***(Terminal date 1)*** *Given history* $(R, X_0, x, P_0, B, b^M, b, Q)$*, date 1 unfolds according to one of the three following situations.*

1. *Monetary dominance: If* $P^F \leq \underline{P}_1$*,* $M$ *sets the date-1 price level at* $\underline{P}_1$ *by setting* $X_1 = \bar{x}\underline{P}_1$*.* $F$ *fully repays maturing bonds:* $l = 0$*, and consumes* $g_1 = \bar{x} + \bar{\tau} - \left(B - b^M P_0/Q + RX_0\right)/\underline{P}_1$*, strictly larger than* $\underline{g}$ *unless* $P^F = \underline{P}_1$ *.*

2. *Fiscal dominance: If* $\underline{P}_1 < P^F \leq \underline{P}_1 + \alpha_M$*,* $M$ *sets the date-1 price level at* $P^F$*.* $F$ *fully repays maturing bonds:* $l = 0$*, and spends at the incompressible level* $g_1 = \underline{g}$*.*

3. *Default: Otherwise,* $M$ *sets the date-1 price level at* $\underline{P}_1$*.* $F$ *fully defaults on* $B$*:* $l = 1$*, and spends* $g_1 = \bar{x} + \bar{\tau} - RX_0/\underline{P}_1 > \underline{g}$*.*

*Proof.* See above. □

Figure 1 illustrates how the date-1 price level $P_1$ evolves as net public liabilities $RX_0 + B - b^M P_0/Q$ increase.

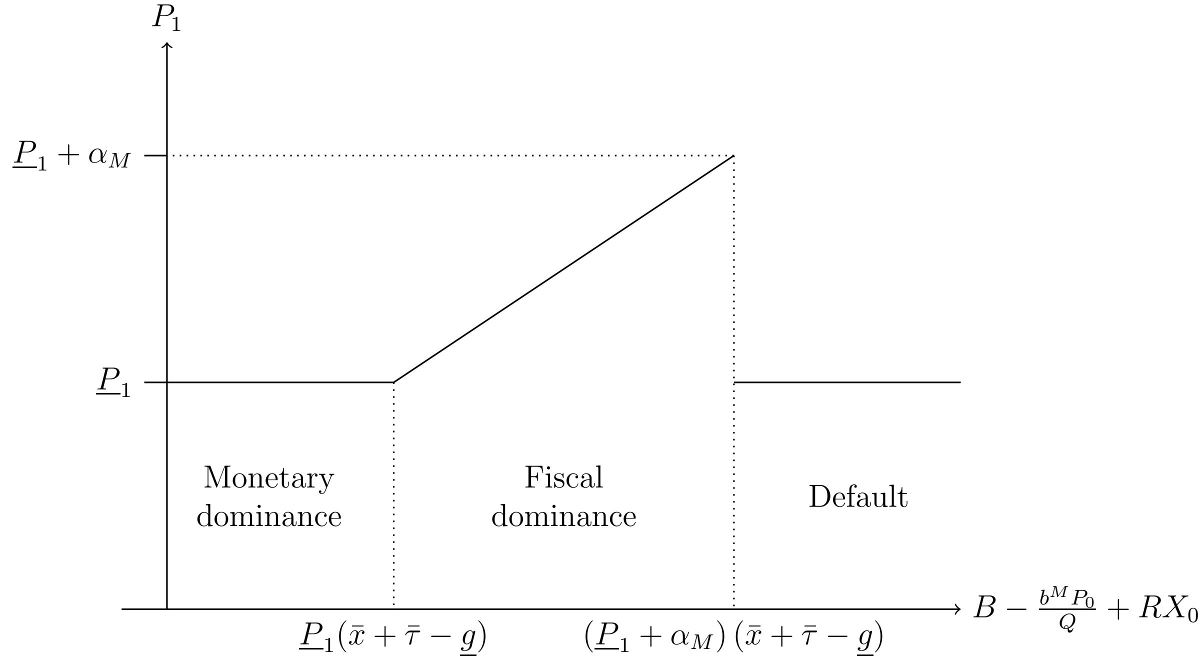


Figure 1: Date-1 price level $P_1$ as a function of net public liabilities held by the private sector $(B - b^M P_0/Q + RX_0)$.

**Comments on "reserve overflow".** In the case of monetary dominance, the only situation in which $M$ might set the price strictly above its date-1 target $P_1^M$ is a fully self-inflicted one independent of fiscal actions, and we deem it one of "reserve overflow". This occurs when the reserves sold by old savers $RX_0$ are strictly larger than $\bar{x}P_1^M$, so that the price level must be at least equal to $RX_0/\bar{x} = \underline{P}_1 > P_1^M$. In this case, $M$ has manufactured its own lower bound on the date-1 price level when deciding on $(R, X_0)$ at date 0, thereby barring itself from reaching its date-1 price level target. We will see below that, given condition (9) and in the absence of a zero lower bound on the policy rate $R$, $M$ can ensure that this does not occur along the equilibrium path, that is, $\underline{P}_1 = P_1^M$ in equilibrium when there is either monetary dominance or default at date 1. We will also see that there exist cases in which $M$ deliberately uses this to commit to a date-1 price level that it finds ex-post excessive (see Proposition 4).

### 3.3 Date-0 government consumption

Having solved for date 1, we now move on to date 0, solving backwards for its various stages. Start with the third stage in which $F$ decides on its consumption. The transfer to the fiscal authority $F$ from the monetary authority $M$ is $\theta_0 = x - R_{-1}X_{-1}/P_0 - b^M$, equal to the resources from reserve issuances $x - R_{-1}X_{-1}/P_0$ net of bond purchases $b^M$. $F$ consumes these resources on top of the amount $b + b^M$ collected in the bond market. $F$ thus consumes $g_0 = x + b - R_{-1}X_{-1}/P_0$, independent of the resources spent by the monetary authority to purchase bonds $b^M$.

### 3.4 Date-0 bond market

In the market for government bonds, $F$ issues $B$ bonds, $M$ invests $b^M$, and then savers invest $b$. From Proposition 1, these actions lead to one of the following date-1 situations: monetary dominance, fiscal dominance, or default. It is easy to see that default cannot be an equilibrium outcome. Since default is total ($l = 1$) when it occurs, savers' rationality would imply $b = 0$ in case of date-1 default, and $F$ would receive (at best) only resources from $M$ in the bond market against an empty promise. But then $F$ would be strictly better off not issuing bonds ($B = 0$) and receiving these resources as a dividend from $M$, as this averts default leaving $g_0$ and $g_1$ unchanged.

**Bond market equilibrium given $B$ and $b^M$.** In the absence of default, if $F$ issues $B$ bonds and $M$ then invests $b^M$, savers' optimal portfolio choice and market clearing yield a bond price $Q$ and savers' investment $b$ such that

$$r = \frac{P_0}{P_1 Q} \text{ and } QB = P_0(b^M + b), \tag{13}$$

where $P_1$ is given by Proposition 1.

We now derive optimal date-0 debt issuance $B$ by $F$ as follows. We first study which debt level grants $F$ the highest date-0 utility among all the levels that lead to date-1 monetary dominance. We then describe the optimal debt level among those that generate date-1 fiscal dominance. Finally, we compare these two conditionally optimal debt levels.

**Monetary dominance.** Suppose first that the bond issuance $B$ by $F$ leads to strict monetary dominance at date 1 ($P_1 = \underline{P}_1 < P^F$). Optimal debt issuance by $F$ requires

$$\max_B \; g_0 + \beta g_1 \tag{14}$$

$$\text{s.t. } \underline{g} \leq g_0 = b + x - \frac{R_{-1}X_{-1}}{P_0}, \tag{15}$$

$$\underline{g} < g_1 = \bar{x} + \bar{\tau} - \frac{B - \frac{b^M P_0}{Q} + RX_0}{\underline{P}_1}, \tag{16}$$

$$B - \frac{b^M P_0}{Q} = rb\underline{P}_1. \tag{17}$$

Date-0 consumption (15) stems from Section 3.3, date-1 consumption (16) from Proposition 1 (with a strict inequality because we consider strict monetary dominance $P_1 = \underline{P}_1 < P^F$), and condition (17) from the bond-market equilibrium relations (13). Notice that combining these latter two equations, this program depends on $B$ and $b^M$ only through (17). This is because $M$ pays as date-0 dividends whichever amount it does not invest in the bond market, and pays as date-1 dividends whichever bond repayment it collects. Thus $F$ can choose the real amount borrowed from savers $b$ by correctly anticipating $b^M$ when selecting the nominal amount $B$, and the value of $b^M$ does not affect any agent's payoff. We therefore restrict without loss of generality the analysis to $b^M = 0$.[9] Notice also that condition (9) ensures that there exists $b \geq 0$ satisfying (15) and (16).

It cannot be that $\beta r < 1$, otherwise $F$ would seek to minimize its date-1 consumption to $g_1 = \underline{g}$ from (14), contradicting strict monetary dominance. Thus a necessary condition

[9]This irrelevance of $b^M$ because it plays no strategic role is a particular version of Wallace (1981) on the irrelevance of open-market operations. By contrast, $b^M$ matters and is uniquely pinned down in the date-1 fiscal-dominance case studied below, in which it is strategically relevant.

for strict monetary dominance is $\beta r \geq 1$. If $\beta r \geq 1$, $F$ maximizes its utility conditional on strict monetary dominance (strictly so if $\beta r > 1$) by borrowing $b^*$, the smallest amount necessary to consume $\underline{g}$ at date 0:

$$b^* \equiv \left(\underline{g} - x + \frac{R_{-1}X_{-1}}{P_0}\right)^+, \tag{18}$$

and this yields $F$ a utility

$$x - \frac{R_{-1}X_{-1}}{P_0} + b^* + \beta\left(\bar{x} + \bar{\tau} - rb^* - \frac{RX_0}{\underline{P}_1}\right). \tag{19}$$

**Fiscal dominance.** Suppose now that the bond issuance $B$ leads to date-1 fiscal dominance: $P_1 = P^F$ and $g_1 = \underline{g}$. In this case, combining the definition of $P^F$ given by (11) and the equilibrium determination of the bond price (13) yields a date-1 price level

$$P_1 = P^F = \frac{B + RX_0}{\bar{x} + \bar{\tau} - \underline{g} + rb^M}. \tag{20}$$

The date-1 price is thus decreasing in $b^M$, and so it must be that $M$ optimally invests as much as possible in the date-0 bond market, that is, $b^M = x - R_{-1}X_{-1}/P_0$. This implies in turn that the date-1 price level is strictly (and linearly) increasing in $B$. Conditionally on date-1 fiscal dominance, $F$'s utility is

$$x - \frac{R_{-1}X_{-1}}{P_0} + \frac{1}{r}\left(\bar{x} + \bar{\tau} - \underline{g} - \frac{RX_0}{P^F}\right) + \beta\underline{g}, \tag{21}$$

strictly increasing in $P^F$. Thus $F$ issues $B$ so that $P^F$ takes the largest possible value that $M$ prefers to forcing default, $\underline{P}_1 + \alpha_M$.

In the remainder of the paper, we deem "price-level taking" debt level the amount of debt that $F$ optimally raises conditionally on expecting date-1 monetary dominance, and the "Sargent-Wallace" debt level the optimal quantity of debt among those that are conducive to date-1 fiscal dominance. From above, the former debt level corresponds to real proceeds $b^*$ whereas the latter is given by setting $P^F = \underline{P}_1 + \alpha_M$ and $b^M = x - R_{-1}X_{-1}/P_0$ in (20). Comparing now the maximum utilities (19) and (21) that $F$ can achieve under each level, simple algebra yields that $F$ prefers the price-level taking debt level if and only if

$$\underbrace{(\beta r - 1)}_{\text{Unit cost of frontloading } g} \times \underbrace{\left(\bar{x} + \bar{\tau} - \underline{g} - rb^* - \frac{RX_0}{\underline{P}_1}\right)}_{\text{Net public resources}} \geq \underbrace{RX_0 \left(\frac{1}{\underline{P}_1} - \frac{1}{\underline{P}_1 + \alpha_M}\right)}_{\text{Fiscal-dominance gains}}. \tag{22}$$

This condition admits a simple interpretation. Relative to the price-level taking debt level, the Sargent-Wallace one generates additional resources from applying a higher inflation on the reserves $RX_0$ held by savers at date 1 (right-hand side of (22)). Generating these resources comes at the cost of frontloading the date-1 consumption of the government, however (left-hand side of (22)). The unit frontloading cost is $\beta r - 1$, and is actually a unit gain if $\beta r \leq 1$, in which case $F$ always prefers the Sargent-Wallace debt level. This unit cost applies to the resources of the public sector $\bar{x} + \bar{\tau}$ net of the date-1 value of its liabilities, both explicit (reserves and bonds) and implicit (incompressible expenditures). $F$ prefers the price-level taking debt level if this cost from the Sargent-Wallace debt level exceeds the benefits. The following proposition summarizes these results.

**Proposition 2.** ***(Debt issuance in the date-0 bond market)*** *Given $(R, X_0, x, P_0)$, $F$ issues one of either debt level:*

- ***Price-level taking debt level:*** *$F$ issues bonds so as to optimize its consumption pattern taking the date-1 price level $\underline{P}_1$ as given: It raises an amount $b^*$ of real resources. $M$'s bond purchases are immaterial. There is no default at date 1.*

- ***Sargent-Wallace debt level:*** *$F$ issues a larger amount in the bond market, front-loading consumption as much as possible ($g_1 = \underline{g}$) and issues enough debt to force a date-1 price level given by fiscal dominance. $M$ buys back as many bonds as possible: $b^M = x - R_{-1}X_{-1}/P_0$, but not the whole issuance. The date-1 price level is equal to $\underline{P}_1 + \alpha_M$. There is no default at date 1.*

*$F$ selects the "price-level taking" debt level whenever*

$$(\beta r - 1)\left(\bar{x} + \bar{\tau} - \underline{g} - rb^* - \frac{RX_0}{\underline{P}_1}\right) \geq RX_0 \left(\frac{1}{\underline{P}_1} - \frac{1}{\underline{P}_1 + \alpha_M}\right). \tag{23}$$

*Proof.* See above. □

The "Sargent-Wallace" debt level whereby $F$ floods the bond market with paper so as to force $M$ to "chicken out" and inflate away outstanding reserves at date 1 in order to ensure public solvency is closely related to that underlying the unpleasant monetarist arithmetic in Sargent and Wallace (1981). $F$ creates a deficit that forces $M$ to generate income in an inflationary way, simply by inflating away the value of reserves here. Proposition 2 shows that this need not be $F$'s favorite strategy as this may induce an excessive distortion of its optimal spending relative to the gains from inflation. We are now equipped to solve for the first stage of the game: the date-0 market for reserves.

### 3.5 Date-0 reserve market

We describe the date-0 reserve market in two steps. Proposition 3 first characterizes situations in which monetary dominance prevails at both dates 0 and 1. Proposition 4 then tackles situations in which $M$ cannot reach this outcome.

**Proposition 3.** ***(Characterization of monetary dominance)*** *The equilibrium is such that price levels are on target at dates 0 and 1 ($P_0 = P_0^M$ and $P_1 = P_1^M$) if and only if*

$$(\beta r - 1)\left(\bar{x} + \bar{\tau} - (1+r)\underline{g} - \frac{R_{-1}X_{-1}}{P_0^M}\right) \geq \frac{rR_{-1}X_{-1}}{P_0^M}\frac{\alpha_M}{P_1^M + \alpha_M}. \tag{24}$$

*If this holds, $M$ issues no or sufficiently small new reserves, and announces a rate $R = rP_1^M/P_0^M$. The game then unfolds as in the price-level taking debt level situation in Proposition 2 with $P_1 = P_1^M$.*

*Proof.* See Appendix A. □

Condition (23) driving the bond issuance of $F$ suggests that $M$ must keep the quantity of reserves $RX_0$ with which it starts out date 1 sufficiently low if it wants to impose monetary dominance at date 1. Accordingly, condition (24) states that $M$ can enforce monetary dominance at dates 0 and 1 ($P_0 = P_0^M$ and $P_1 = P_1^M$) if the legacy reserves $R_{-1}X_{-1}$ are sufficiently small other things being equal. In this case, by issuing no new reserves, or a sufficiently small amount of them, $M$ makes the gains from the Sargent-Wallace debt level sufficiently small that $F$ does not issue it. $M$ is indifferent between several level of reserves below a threshold (unless (24) binds) because reserves and bonds are perfect substitutes, and so the resources that $M$ raises and transfers to $F$ to fund $g_0$ can be raised by $F$ at the same cost in the bond market.

In addition to low legacy public liabilities $R_{-1}X_{-1}$, the other interesting feature that drives monetary dominance is the existence of a large fiscal space $\bar{x}+\bar{\tau}-(1+r)\underline{g}$. In this case, $F$ needs to engineer a very large distortion of its public finances in the form of large current borrowing and spending in order to be credibly ready to default in the future. It is important at this point to recall that the analysis is carried out under condition (8) ensuring that $F$ does not contemplate default as long as it can consume at least $\underline{g}$ without taxing more than $\bar{\tau}$. Thus the case in which $F$ has a lot of fiscal space is also one in which $F$ has a sufficiently large aversion to default.

If $F$ has limited fiscal space or/and there are large legacy liabilities so that inequality (24) fails to hold, then $F$ may find it preferable to double down and worsen its situation so as to force help from the monetary authority by issuing the Sargent-Wallace debt level. The following proposition describes date-0 monetary policy in this case.

**Proposition 4.** ***(Optimal monetary policy without monetary dominance)*** *Suppose that condition* (24) *in Proposition 3 does not hold. $M$ adopts one of the following three strategies in the reserve market:*

1. *$M$ announces a rate $R=r(P_1^M+\alpha_M)/P_0^M$ and is indifferent between several levels of newly created reserves (including 0). The date-0 price level is $P_0^M$ and then the game unfolds according to the Sargent-Wallace debt level situation with $P_1=P_1^M+\alpha_M$.*

2. *$M$ announces a rate $R=rP_1^M/P_0$, where $P_0>P_0^M$ and issues no new reserves ($X_0=R_{-1}X_{-1}$). Then the game unfolds according to the price-taking debt level situation with $P_1=P_1^M$.*

3. *$M$ announces a rate $R=rP_1/P_0$, where $P_0\geq P_0^M$ and $P_1>P_1^M$. It issues reserves $P_0\bar{x}/r-R_{-1}X_{-1}\geq 0$. Then the game unfolds according to the price-taking debt level situation with $P_1=RX_0/\bar{x}>P_1^M$ (reserve overflow).*

*Furthermore, strategy 1 prevails if $\beta r\leq 1$, and strategy 2 prevails if $\beta r>1$ and $R_{-1}X_{-1}$ is sufficiently small other things being equal.*

*Proof.* See Appendix A. □

In strategy 1, $M$ "surrenders" and does not try to deter $F$ from issuing the Sargent-Wallace debt level. This is the only strategy in which $M$ is indifferent between several reserve issuance levels whose range is detailed in the proof of Proposition 4.

In strategies 2 and 3, by contrast, $M$ deters $F$ with a strategic use of both interest rate and quantity of reserves. In strategy 2, $M$ reduces the real value of legacy reserves at date 0 by announcing a low interest rate that sets $P_0$ above target, and issues no new reserves. Formally, $M$ sets the date-0 price level at the smallest value such that (24) holds. This strategy both reduces the basis $R_{-1}X_{-1}/P_0$ to which the Sargent-Wallace induced rate of "seigniorage" $\alpha_M/P_1^M$ applies (right-hand side of (24)), and creates fiscal space that $F$ must eliminate at a cost to create such seigniorage (left-hand side of (24)). In sum, strategy 2 is one of preemptive inflation meant to avoid larger future inflation.

In strategy 3, $M$ combines shrinking this way the basis $R_{-1}X_{-1}/P_0$ to which the rate of seigniorage $\alpha_M/P_1^M$ applies by setting $P_0 \geq P_0^M$ together with a reduction in this seigniorage rate by setting $P_1 > P_1^M$. Committing to a date-1 price level above target requires however that $M$ creates its own future lower bound by issuing new reserves at date 0. This expansion of reserves is costly for the same reasons why not issuing new reserves is optimal in strategy 2.

Which of these three strategies is optimal depends on the parameters in a generally complex fashion. The analysis is tractable in two important cases stated in the proposition. First, $M$ has no choice but going for strategy 1 when $\beta r \leq 1$. In this case, $F$ finds frontloading consumption optimal even when holding the date-1 price level fixed. It is thus always happy to issue enough nominal debt against this date-0 consumption that it can get additional resources along the way by forcing $M$ to go beyond its target at date 1.

Second, strategy 2 of preemptive inflation is optimal if $\beta r > 1$ and $R_{-1}X_{-1}$ is sufficiently small. Compare it first to strategy 1. The latter comes at a fixed utility cost $\beta_M \alpha_M$ for $M$. Conversely, the cost of strategy 2 is linearly increasing in $R_{-1}X_{-1}$. In particular if $R_{-1}X_{-1}$ is sufficiently small other things being equal that the economy is not too far off from condition (24), the rise in $P_0$ that warrants the price-level taking strategy is sufficiently small that it induces a disutility $P_0 - P_0^M < \beta_M \alpha_M$. Compare now strategies 2 and 3. The latter consists in raising $P_1$ on top of raising $P_0$. This requires the issuance of new reserves such that $X_0 = P_0 \bar{x}/r$ in order to create a reserve overflow at date 1. This level of new reserves creates a fixed cost—making condition (23) harder to satisfy, smaller than the benefits from being able to raise $P_1$ a little bit over $P_1^M$, which is all that is needed for $R_{-1}X_{-1}$ sufficiently small.

### 3.6 Discussion

**Ex-ante fiscal gains from the unpleasant arithmetic.** It is worthwhile stressing that $F$ does not derive ex-ante gains from issuing the Sargent-Wallace debt level when it does so in equilibrium. When it finds it optimal to do so ex-post, it is anticipated in the reserve and bond markets, so that all public liabilities command the same real return $r$. $F$ on the other hand incurs the costs from excessive borrowing when $\beta r > 1$. In this case, $F$ would be happy to avail itself of a commitment device to not issue at the Sargent-Wallace level, such as a credible fiscal requirement putting an upper bound on the amount of debt it can issue.

There are also parameter values such that $F$ derives ex-ante gains from its ex-post optimal behavior. These correspond to the equilibria in which $M$ deters the Sargent-Wallace debt level with an increase in $P_0$—in strategy 2 and possibly (but not necessarily) in strategy 3. This erodes the value of the legacy liabilities, thereby generating additional public resources for consumption. Furthermore, $F$ does not borrow inefficiently in this case and thus extracts these benefits at no cost.[10]

**What if $F$ is financially constrained?** Condition (8) implies that $F$ is financially unconstrained in the sense that it can borrow against its entire future resources $\bar{x} - RX_0/P_1 + \bar{\tau} - \bar{g}$. Thus the default boundary that it must reach when entering into the Sargent-Wallace debt level is equal to the point at which it would be forced to either raise taxes above $\bar{\tau}$ or cut expenditures below $\underline{g}$ in order to make good on its debt. This situation in which borrowing constraints play no role is a natural first step. The main insights are identical, however, if $F$ is financially constrained. Suppose that condition (8) is replaced with

$$r\underline{g} \leq \alpha_F < \bar{\tau} - \underline{g}, \tag{25}$$

so that $F$ cannot borrow against its entire future resources, but can borrow enough to fund date-0 incompressible expenditures $\underline{g}$. In this case, the default boundary is hit when $F$ owes real debt $\alpha_F$ at date 1, as it finds default preferable to cutting spending by $\alpha_F$ in this case. The counterpart of condition (22) under which $F$ prefers the price-level taking

[10]This should however be anticipated in the unmodelled date-(-1) reserve market in which $R_{-1}X_{-1}$ is issued.

debt level is in this case[11]

$$\underbrace{(\beta r - 1)}_{\text{Unit cost of frontloading } g} \times \underbrace{\left(\frac{\alpha_F}{r} - b^*\right)}_{\text{Amount to be frontloaded}} \geq \underbrace{\beta R X_0 \left(\frac{1}{\underline{P}_1} - \frac{1}{\underline{P}_1 + \alpha_M}\right)}_{\text{Fiscal-dominance gains}}. \tag{26}$$

The only difference with condition (22) is that the Sargent-Wallace debt level no longer corresponds to borrowing against the entire date-1 resources net of incompressible expenditures, but only against the default boundary $\alpha_F$.[12] Condition (26) shows that a higher cost of default makes the Sargent-Wallace debt level more costly and thus less appealing to $F$.

**What if $F$ can take over monetary policy?** This paper posits that the central bank is independent in the sense that it has its own objectives, and has a free hand at managing its balance sheet to pursue them. We study whether the government can impose its views on it by means of fiscal tools. The government could also avail itself of other means, such as the appointment of cronies at the helm of the bank, or such as legal reforms reneging on independence. In such extensions, which are beyond the scope of this paper, one would have to compare the costs of these strategies to that of Sargent-Wallace expansions in order to determine the fiscal authority's equilibrium behavior. It is important to stress that in addition to the obvious political costs from bringing the monetary authority to heel this way, which are presumably large in jurisdictions in which central-bank independence has prevailed for several decades, the fiscal authority would also find it more difficult to borrow ex-ante if the market perceives the fiscal authority's ex-post costs from high inflation to be low.

**What if the bond market opens before the reserve market?** Suppose that the bond market opens and clears before that for reserves at date 0. The insights are broadly similar to that when $M$ issues reserves first.[13] The main difference is that $F$ cannot benefit from forcing a date-1 price level above target by borrowing a lot at date 0 since this would be anticipated in both date-0 bond and reserve markets. $F$ may however still find it worthwhile forcing $M$ to set the date-0 price level at $P_0^M + \alpha_M$ so as to reduce the

[11] We omit the derivation for brevity, it is available upon request.

[12] The date-0 expenditures $b^*$ are substracted from this level because $F$ has to borrow to fund them anyway in the price-level taking strategy.

[13] The full analysis is available upon request.

date-0 real value of legacy reserves $R_{-1}X_{-1}$. This is so again when the associated gain more than offsets the cost from excessive date-0 borrowing. But then, the interesting analysis of optimal monetary policy in anticipation of this behavior—the equivalent of Propositions 3 and 4—would have to take place in the date-(-1) reserve market at which these reserves are issued.

**Return on central bank investments.** One can interpret $\bar{x}$ as including not only an exogenous demand for reserves but also the return on investments that $M$ funded with the proceeds from issuing $X_{-1}$ at date $-1$. This implies that monetary dominance benefits from a high expected return viewed from date 0. This shapes the risk-taking incentives of $M$ when investing at date -1. In particular, if fiscal dominance is very likely viewed from date $-1$ conditionally on investing in safe assets, $M$ may be tempted to opt for assets with riskier returns to increase the probability of monetary dominance. Such gambling for resurrection behavior would parallel that of investors subject to limited liability constraints as studied in the finance literature (see Allen and Gale, 2000, among others).

We now extend this baseline model in two directions. Section 4 first posits standard convex costs of taxation. Section 5 then studies a simple model of endogenous (real) interest rate. In both extensions, unlike in the baseline model, a low equilibrium interest rate ($r < 1/\beta$) is no longer a sufficient condition for fiscal dominance. Monetary dominance may prevail at arbitrarily low equilibrium interest rates if the out-of-equilibrium costs from repaying the Sargent-Wallace debt level at date 1 is too large, either because of high taxation costs (Section 4) or of a high interest rate (Section 5). This distinction between in and out of equilibrium costs of debt is moot in the baseline model in which the marginal cost of taxation (zero below $\bar{\tau}$) and the interest rate are constant across debt levels.

# 4 General cost of taxation

The main simplification in the baseline model is a marginal cost of taxation that jumps from 0 to an arbitrarily large value at $\bar{\tau}$, leading to simple final default and taxation decisions by the fiscal authority. This section posits smooth convex taxation costs. This more general specification enables us to both confirm and sharpen the broad insights of the baseline model. In particular, we obtain that, depending on $F$'s aversion to default, the level of debt that pushes $M$ to chicken out may lead the level of taxes above that under monetary dominance; this implies a large distortionary cost of taxation under fiscal dominance, so that $F$ prefers to comply with monetary dominance.

We substitute Assumption 1 with the following set of assumptions:

**Assumption 2.** ***(General cost of taxation)***

- *The cost of taxation $c$ is such that $c'$ exists and is an increasing bijection over $[0;+\infty)$.*
- *$F$'s instantaneous utility from consumption is $v(g) = g$.*

Besides introducing a general convex cost of taxation, we also assume away the existence of an incompressible level of expenditures for the sake of simplicity ($\underline{g} = 0$). As with the study of the baseline model, we proceed backwards, focussing the exposition on the features of the equilibrium that depart from that in the baseline model.

## 4.1 Date-1 taxation and default decisions

The program that $F$ solves after the date-1 reserve market has cleared is

$$\max_{l\in[0,1],\tau\geq 0}\left(\bar{x}+\tau-\frac{RX_0+(1-l)B}{P_1}+\frac{(1-l)b^M P_0}{P_1 Q}\right)-c(\tau)-\mathbb{1}_{\{l>0\}}\alpha_F, \tag{27}$$

$$\text{s.t. } \bar{x}+\tau-\frac{RX_0+(1-l)B_0}{P_1}+\frac{(1-l)b^M P_0}{P_1 Q}\geq 0. \tag{28}$$

The fixed default cost implies that as in the baseline model, $F$ either repays $B$ in full ($l = 0$) or fully defaults ($l = 1$). Let us introduce

$$\tau^* \equiv \arg\max\{\tau - c(\tau)\} = (c')^{-1}(1) \tag{29}$$

the taxes that $F$ optimally raises at date 1 if it does not need to tax more to be solvent. $F$ then prefers to repay its bond if and only if

$$\bar{x}+\tau_1-\frac{RX_0+B}{P_1}+\frac{b^M P_0}{P_1 Q}-c(\tau_1)\geq \bar{x}+\tau^*-\frac{RX_0}{P_1}-c(\tau^*)-\alpha_F, \tag{30}$$

where $\tau_1$ is the optimal level of taxes conditional on repayment, satisfying

$$\tau_1 \equiv \max\left\{\frac{RX_0 + B}{P_1} - \frac{b^M P_0}{P_1 Q} - \bar{x}; \tau^*\right\}. \tag{31}$$

Rearranging (30) as follows offers a natural interpretation:

$$\underbrace{c(\tau_1) - \tau_1 - c(\tau^*) + \tau^*}_{\text{Relative disutility of taxation}} \leq \underbrace{\alpha_F - \left(\frac{B}{P_1} - \frac{b^M P_0}{P_1 Q}\right)}_{\text{Net cost of default}}. \tag{32}$$

Taxes $\tau_1$ when making good on debt are by definition (31) weakly higher than that when defaulting, equal to $\tau^*$. The relative net utility cost of taxation (differential taxation cost minus proceeds on the left-hand side of (30)) is then positive. On the right-hand side of (30), the net utility cost of default is the fixed cost $\alpha_F$ net of the gains from defaulting on the debt held by private agents $B_0/P_1 - b^M P_0/P_1 Q$. Overall, $F$ repays $B$ when the disutility from taxation when repaying relative to that when defaulting $(c(\tau_1) - \tau_1 - c(\tau^*) + \tau^*)$ is low, the fixed cost of default $(\alpha_F)$ is large, or public debt net of central bank's holdings $(B_0/P_1 - b^M P_0/(P_1 Q))$ is small.

## 4.2 Date-1 price level

Other things being equal, an increase in the date-1 price level $P_1$ reduces both the relative cost of taxation when repaying and the gains from defaulting, and so it makes repayment more appealing to $F$. The cost of taxation decreases in $P_1$ because so does $\tau_1$ from (31). The gain from default decreases in $P_1$ because so does the real repayment due.

As in the baseline model, we let $P^F$ denote the minimum price level that ensures that $F$ is willing to repay—the value of $P_1$ such that condition (30) holds as an equality. Notice that an explicit formula for $P^F$ such as (11) in the baseline model is out of reach.

Notice also that net public debt and reserves do not affect $P^F$ symmetrically here as they do in the baseline model in which their sum determines $P^F$ (see expression (11)). Here more debt not only increases the distortionary cost of taxes in the case of repayment—as is symetrically the case for more reserves, but it also increases the gain from defaulting. This latter effect is absent in the baseline model in which the assumed discontinuity in the marginal cost of taxation implies that the fiscal authority has a strict preference for not defaulting at $P_1 = P^F$.

As in the baseline model, $M$ compares $P^F$ to $\underline{P}_1 = \max\left\{RX_0/\bar{x}, P_1^M\right\}$ and to $\underline{P}_1+\alpha_M$. This leads to monetary dominance when $P^F \leq \underline{P}_1$, in which case the price level at date 1 is $P_1 = \underline{P}_1$, to fiscal dominance when $\underline{P}_1 < P^F \leq \underline{P}_1 + \alpha_M$, in which case $P_1 = P^F$, and to default otherwise, in which case $P_1 = \underline{P}_1$.

## 4.3 Date-0 bond market

Date-0 government consumption is verbatim that in the baseline model (with $\underline{g} = 0$), and so we turn to the date-0 bond market. For the same reason as in the baseline model, there is no default in equilibrium, and a given debt issuance by $F$ leads either to monetary or fiscal dominance at date 1. As in the baseline model, we study optimal debt issuance conditional on either date-1 outcome.

**Monetary dominance.** Among all "price-level taking" debt levels, the optimal one is $B = \underline{P}_1 r b^{PT}$, where $b^{PT}$ solves:

$$\max_{b\geq 0} \{g_0 + \beta g_1 - \beta c(\tau_1)\} \tag{33}$$

$$\text{s.t. } \tau_1 = \max\left\{\frac{RX_0 + B}{\underline{P}_1} - \frac{b^M P_0}{\underline{P}_1 Q} - \bar{x}; \tau^*\right\}, \tag{34}$$

$$g_0 = x + b - \frac{R_{-1}X_{-1}}{P_0}, \tag{35}$$

$$g_1 = \bar{x} + \tau_1 - \frac{RX_0}{\underline{P}_1} - rb, \tag{36}$$

$$c(\tau_1) - \tau_1 - c(\tau^*) + \tau^* \leq \alpha^F - rb, \tag{37}$$

$$g_0 \geq 0. \tag{38}$$

As in the baseline model, purchases of bonds by $M$ are immaterial under monetary dominance, and so we assume without loss of generality $b^M = 0$ in this program. The optimal debt level critically depends on the level of the interest rate $r$. When $\beta r \geq 1$, $F$ does not borrow and the level of taxes is at its unconstrained maximum $\tau_1 = \tau^*$. When $\beta r < 1$, $F$ borrows as much as it can against its date-1 resources : $b$ is selected so that $g_1 = 0$. The date-1 taxes driving these date-1 resources are the minimum of two values, either $(c')^{-1}(1/\beta r)$ or the solution in $\tau_1$ to {(36);(37)} with $g_1 = 0$ in (36). In the former case, which prevails if $\alpha_F$ is sufficiently large other things being equal, $F$ strictly prefers to make good on its debt at date 1 whereas it is indifferent in the latter case.

**Fiscal dominance.** Suppose now that $F$ issues debt $B$ so that the date-1 outcome is fiscal dominance. In this case, the date-1 taxes $\tau_1$, the date-1 price-level $P^F$, and savers' investment in the date-0 bond market $b$ solve the three equations:

$$\tau_1 = \max\left\{\frac{RX_0}{P^F} + rb - \bar{x}; \tau^*\right\}, \tag{39}$$

$$c(\tau_1) - \tau_1 - c(\tau^*) + \tau^* = \alpha_F - rb, \tag{40}$$

$$B = r\left(b + x - \frac{R_{-1}X_{-1}}{P_0}\right)P^F. \tag{41}$$

The first two equations state that $F$ must be indifferent between defaulting or making good on $B$ at at date 1, and the third one expresses bond-market clearing. These equations take into account that investors in bonds correctly anticipate that $P_1 = P^F$, and that $M$ optimally invests as much as possible in the bond market ($b^M = x - R_{-1}X_{-1}/P_0$).

The solution to this system is such that $P^F$ and $b$ increase with respect to $B$ whereas $\tau_1$ decreases. Suppose otherwise that $b$ decreases in $B$. Equation (41) implies that $P^F$ must increase, but then $\tau_1$ must decrease from (39) and increase from (40), a contradiction. So, $b$ increases in $B$, (40) implies that $\tau_1$ decreases, and (39) in turn that $P^F$ increases.

Since increasing $B$ both raises $P^F$, thereby eroding the value of reserves $RX_0$, and reduces taxes $\tau_1$, $F$ finds it optimal, as in the baseline model, to set $B$ as large as possible up to the point at which $P^F = \underline{P}_1 + \alpha_M$.

## 4.4 Date-0 reserve market

The generic result shown in the baseline model that $M$ seeks to discourage $F$ from issuing the Sargent-Wallace debt level by keeping the amount of circulating reserves sufficiently low still holds. The detailed analysis of monetary policy carried out in the case of the baseline model is however more cumbersome in this case. For brevity, we skip it here, and only state the most interesting result showing that the central role of the interest rate in the baseline model owed to the very simple assumed cost of taxation.

**Proposition 5.** ***(Monetary dominance can prevail when*** $\beta r < 1$***.)*** *Suppose* $\bar{x}/r \geq R_{-1}X_{-1}/P_0^M$*. If other things being equal* $\alpha_F$ *is sufficiently large, then the price level is on target at every date (*$P_0 = P_0^M$ *and* $P_1 = P_1^M$*).*

*Proof.* See Appendix B. □

Recall that in the baseline model, an interest rate $r$ strictly smaller than $1/\beta$ warrants that $F$ issues the Sargent-Wallace debt level. It is not the case here, as a sufficiently large cost of default $\alpha_F$ ensures monetary dominance given the value of the interest rate.

The reason is that if $F$ cares a lot about solvency ($\alpha_F$ sufficiently large other things being equal), then it must credibly make the future fiscal cost of avoiding default very large too, and thus it must issue a lot of debt forcing large future taxes—to a level that exceeds the one under monetary dominance. Whereas a low interest rate ($\beta r < 1$) makes borrowing against given date-1 fiscal resources appealing, producing very large such future resources out of costly taxation is not. Thus, is $\alpha_F$ is sufficiently large, even though $F$ borrows against its entire future taxes in equilibrium ($g_1 = 0$), it still strictly prefers taxation over default at date 1, and so monetary dominance prevails.

In the baseline model, by contrast, the stark assumption about taxation costs means that date-1 taxes are fixed at $\bar{\tau}$ no matter date-0 strategies. In this case, the cost from frontloading this fixed tax income is always negative when $\beta r < 1$, making the Sargent-Wallace debt level dominant in this case.

In sum, this more general specification for taxation costs shows that it is not the equilibrium value of the interest rate that determines the dominance regime, but rather the (possibly out-of-equilibrium) cost from issuing the amount of debt required for the Sargent-Wallace debt level. Taxation costs are part of these costs. The following extension with an endogenous interest rate introduces another source of such costs in the form of an increase in the interest rate.

# 5 Variable interest rate

This section studies an extension of the baseline model in which the issuance of public liabilities affects the interest rate. Formally, we modify the baseline model as follows.

**Assumption 3.** ***(Variable-rate model)***

- *Savers are endowed with one consumption unit at date 0, and with a large quantity of them at date 1. Their preferences are given by $u(c_0) + c_1/r$, where $u'$ exists and is a decreasing strictly convex bijection mapping $(0, 1]$ into $[u'(1), +\infty)$.*

- *We drop condition* (9)*.*

- *As in the baseline model, taxes come at no cost up to the threshold* $\bar{\tau} \geq 0$*, and at an arbitrarily large cost beyond it.*
- *For notational simplicity, we assume that* $\underline{g} = 0$*.*

Here the public sector lifts the (real) interest rate when issuing liabilities simply because it reduces savers' date-0 consumption. This impact on the interest rate could stem in practice from other mechanisms such as the crowding out of private investment.[14] Condition (9) is no longer relevant as it involves a fixed assumed discount rate $r$. For all $x \in [0, 1)$, we define

$$r(x) \equiv ru'(1 - x). \tag{42}$$

The backwards solution to the game goes as follows. First, it is easy to see that the analysis of date 1 following a history $(R, X_0, x, P_0, B, b^M, b, Q)$ is verbatim that of the baseline model summarized in Proposition 1. The reason is simply that the interest rate no longer plays a role once public liabilities have been issued at date 0. Date-0 spending by $F$ is also identical for the same reason.

Consider now the date-0 bond market given history $(R, X_0, x, P_0)$. $F$ first issues $B$ bonds, and then $M$ invests $b^M$ and savers $b$. For the same reason as in the baseline model, there is no default along the equilibrium path. In the absence of default, bond-market clearing and savers' rationality yield bond price $Q$ and savers' investment $b$ given issuance $B$ and $M$'s bond purchase $b^M$:

$$QB = P_0(b + b^M) \text{ and } \frac{P_0}{P_1 Q} = r(1 - b - x). \tag{43}$$

$F$ anticipates that its bond issuance will lead either to monetary or fiscal dominance at date 1. As in the baseline model, we solve for the optimal debt level conditional on each of these date-1 outcomes.

**Monetary dominance.** The fiscal authority $F$ seeks to optimally consume taking the date-1 price level as given, and thus issues the "price-level taking" debt level $B =$

[14] Crowding out of private investment was actually the force at play in an earlier draft.

$\underline{P}_1 r(1 - b^{PT} - x)b^{PT}$, where

$$b^{PT} \equiv \arg\max_b \{g_0 + \beta g_1\} \tag{44}$$

$$\text{s.t. } g_0 = x + b - \frac{R_{-1}X_{-1}}{P_0}, \tag{45}$$

$$g_1 = \bar{x} + \bar{\tau} - \frac{RX_0}{\underline{P}_1} - r(1 - x - b)b, \tag{46}$$

$$0 \leq b < 1 - x, 0 \leq g_1. \tag{47}$$

As in the baseline model, $b^M$ is payoff irrelevant, and we set it to 0 without loss of generality. Unlike in the baseline model, the convexity of the interest rate schedule $r(.)$ leads to a consumption-smoothing motive between dates 0 and 1. We let $(g_0^{PT}, g_1^{PT})$ denote the consumption stream of $F$ resulting from this program. This corresponds (in the case of an interior solution) to the blue point $(g_0^{PT}, g_1^{PT})$ in Figure 2.[15]

**Fiscal dominance.** A second option for the fiscal authority is to issue debt so that there is fiscal dominance at date 1: The date-1 price level $P_1$ satisfies $P_1 = P^F > \underline{P}_1$, where $P^F$ is given by (11). Fiscal dominance implies that $F$ cannot consume at date 1 from Proposition 1 given $\underline{g} = 0$. Thus, denoting $(g_0^{SW}, g_1^{SW})$ the optimal consumption pattern that $F$ can obtain conditionally on date-1 fiscal dominance, it must be that $g_1^{SW} = 0$ and that $g_0^{SW}$ maximizes date-0 consumption over all the debt levels leading to date-1 fiscal dominance. The proposition below states that the fiscal authority, as in the baseline model, selects the "Sargent-Wallace" debt level such that the date-1 price level is $\underline{P}_1 + \alpha_M$, the largest value of $P^F$ that does not trigger default.

The following proposition summarizes these results.

**Proposition 6.** ***(Debt issuance in the date-0 bond market)*** *Given* $(R_0, X_0, x, P_0)$, *$F$ issues one of either debt level:*

- ***Price-level taking debt level:*** *$F$ issues bonds so as to optimize its consumption pattern taking the date-1 price level $\underline{P}_1$ as given. In this case, $F$ raises an amount $b^{PT}$ of real resources. $M$'s bond purchases are immaterial. There is no default at date 1.*

[15] We are grateful to Vladimir Asriyan for suggesting this graphical representation of our results.

- ***Sargent-Wallace debt level:*** *$F$ issues a larger amount in the bond market, front-loading consumption as much as possible ($g_1^{SW} = 0$) and raises a real amount $b^{SW} \geq b^{PT}$, so as to force a date-1 price level given by fiscal dominance. $M$ buys back as many bonds as possible: $b^M = x - R_{-1}X_{-1}/P_0$, but not the whole issuance. The date-1 price level is above target, equal to $\underline{P}_1 + \alpha_M$. There is no default at date 1.*

*$F$ selects the "price-level taking" debt level whenever*

$$\Delta \equiv g_0^{PT} + \beta g_1^{PT} - g_0^{SW} \geq 0. \tag{48}$$

*Proof.* See Appendix C. □

This Sargent-Wallace debt level and the associated government consumption is depicted by the red point on Figure 2. That $g_1^{SW} = 0$ of course means that this point is on the $x$-axis. The gain in terms of resources for the public sector associated with a price level $P^F$ larger than $\underline{P}_1$ implies that this red point is to the right of the intersection of the $x$-axis with the feasibility frontier in the case of the price-level taking debt level.

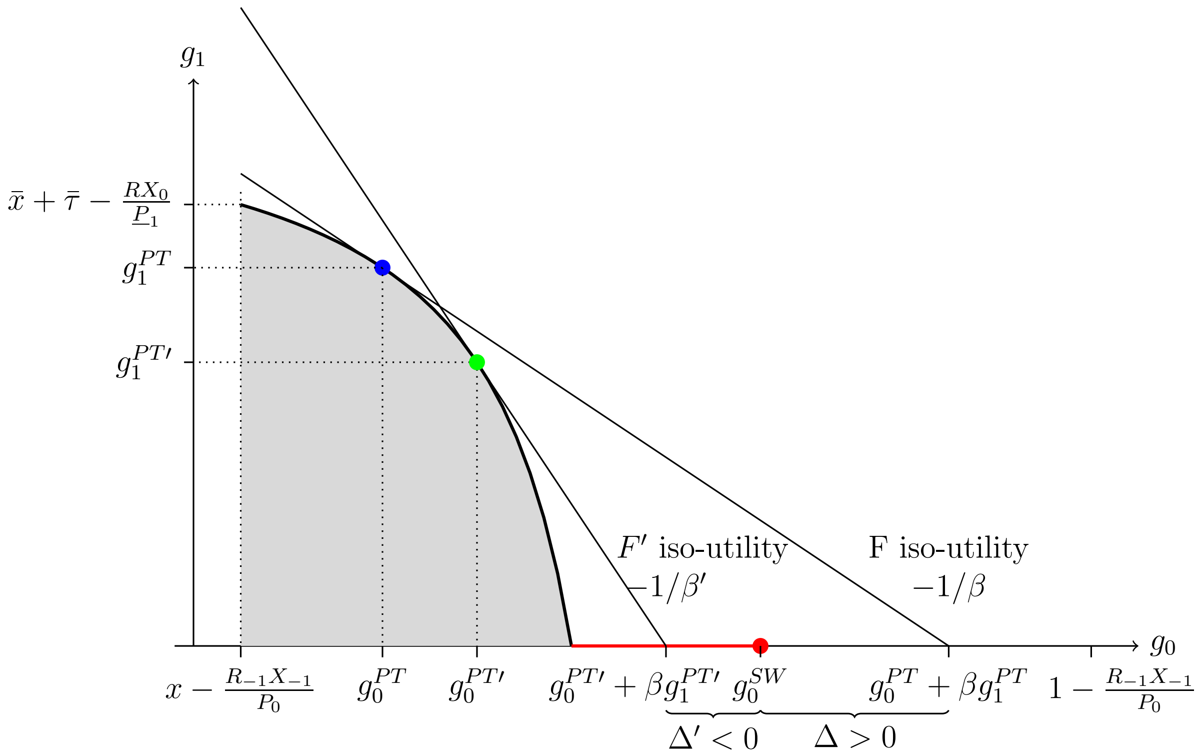


Figure 2: Problem faced by $F$ on the date-0 debt market.
The red circle corresponds to consumption associated with Sargent-Wallace debt issuance. The blue circle corresponds to consumption pattern associated with the price level taking debt level with high $\beta$ and the green circle with low $\beta' < \beta$.

The value of $\Delta$ defined in (48) drives $F$'s issuance decision, and can simply be observed on Figure 2: it corresponds to the (signed) distance along the x-axis between the red point which corresponds to the payoff from the Sargent-Wallace debt level and the intersection between the x-axis and the iso-utility associated with the consumption pattern $(g_0^{PT}, g_1^{PT})$ obtained through the price-taking debt level. Figure 2 displays two situations, one in which $F$ prefers the price-level taking debt level, and one in which $F$ is more impatient ($\beta' < \beta$) and prefers the Sargent-Wallace debt level. In sum, the sign of $\Delta$ measures as in the fixed-rate model the cost of distorted spending net of the gains from inflating reserves $RX_0$.

**Date-0 reserve issuance.** The final step is the determination of the action of $M$ in the date-0 market for reserves. The following proposition is the counterpart when the interest rate is variable of Proposition 3 that spelled out the conditions for monetary dominance at all dates when the rate is fixed. We denote $(g_0^{PT}(0), g_1^{PT}(0))$ the solution to $F$'s optimal spending problem under monetary dominance (14) when $x = R_{-1}X_{-1} = RX_0 = 0$. Notice that this solution is mathematically well defined but not economically so as $M$ needs arbitrary small reserves to pin down the price level.

**Proposition 7.** ***(The determinants of monetary dominance)***

*If $g_1^{PT}(0) > 0$, there exists a threshold $\overline{RX} > 0$ such that if $R_{-1}X_{-1} \leq \overline{RX}$, the unique equilibrium is such that the price level is on target at each date— $P_0 = P_0^M$ and $P_1 = P_1^M$, and such that $M$ minimizes the amount of reserves in circulation ($X_0 = R_{-1}X_{-1}$).*

*If $g_1^{PT}(0) = 0$, any equilibrium is such that $F$ issues the Sargent-Wallace debt level implying $P_1 = \underline{P}_1 + \alpha_M$. $M$ (and thus $F$) is indifferent across several levels of reserves $X_0$.*

*Proof.* See Appendix D. □

Proposition 7 offers two insights. First, it exhibits conditions under which $M$ reaches its price-level objective at each date. As in the fixed-rate case, the first of these conditions is that legacy reserves be sufficiently small. The second one is that $F$ finds frontloading consumption sufficiently costly in the sense that $g_1^{PT}(0) > 0$.

The second insight is that this latter condition is actually necessary: The fiscal authority always enters into the Sargent-Wallace debt level when it fails to hold. The situation in which $g_1^{PT}(0) = 0$ is therefore the counterpart of $\beta r < 1$ in the baseline model, as $F$ enjoys (ex-post) benefits but incurs no cost form the Sargent-Wallace debt level in both cases.

**Equilibrium interest-rate level versus demand curve for public securities.** The most interesting difference between the baseline model and this variable-rate extension is that monetary dominance can prevail at any equilibrium value of the interest rate level, including when it is smaller than $1/\beta$. If the debt issuance required to force $M$ to chicken out triggers a sufficiently large increase in the interest rate (formally, if $g_1^{PT}(0) > 0$ because $r'$ is sufficiently large other things being equal), then monetary dominance can prevail even when the interest rate observed in equilibrium is arbitrarily low.

# 6 Infinite-horizon model

This section studies an infinite-horizon version of the model in which infinitely lived fiscal and monetary authorities interact with a private sector populated by overlapping generations of savers each identical to that in the two-date model. The motive behind this OLG modelling choice is our intent to focus on a dynamically inefficient economy. The infinite horizon would not add significant insights to the two-date setup in the dynamically efficient case. By contrast, when the public sector finances its resources with Ponzi schemes, market forces become the central driver of the price level. This section illustrates this by showing that the key exogenous variables $(\bar{x}, \bar{\tau}, \alpha_M, \alpha_F)$ of the two-date baseline model can all be endogenized as part of the private sector's strategy in the infinite-horizon model.

## 6.1 Setup

Time is discrete and indexed by $t \in \mathbb{N}$.

**Private sector.** At each date $t$, a unit mass of savers are born. They live for two dates and have preferences $c_t + c_{t+1}/r_t$, where $r_t > 0$. They each receive an endowment of the consumption good when young.[16] This economy is dynamically inefficient in the sense

[16] They may also receive consumption units when old but this is immaterial.

that the endowment of cohort $t+1$ is at least $r_t$ times that of cohort $t$.[17]

**Public sector.** The public sector is populated by infinitely-lived monetary and fiscal authorities very much identical to that in the two-date model, except that the fiscal one has no taxation power (more on this below). The extensive form of the game at each date $t$ is similar to that of date 0 in the two-date game. We detail it again as follows.

**Date-t market for reserves.**

1. $M$ selects total date-t outstanding reserves $X_t \geq R_{t-1}X_{t-1}$ by issuing new reserves $X_t - R_{t-1}X_{t-1}$ on top of $R_{t-1}X_{t-1}$ sold by old savers, and announces the interest rate $R_t \geq 0$ between dates $t$ and $t+1$.

2. Young savers invest an aggregate quantity $x_t \geq 0$ of consumption units in the market for reserves at the price $P_t$.

**Date-t bond market.**

3. $F$ issues $B_t \geq 0$ bonds.

4. $M$ invests $b_t^M \in [0, (X_t - R_{t-1}X_{t-1})/P_t]$ consumption units in the bond market.

5. Young savers invest $b_t \geq 0$ aggregate consumption units in the bond market at the price $Q_t$.

**Date-t spending and default.**

6. $F$ decides on the haircut $l_t \in [0,1]$ on legacy debt $B_{t-1}$ and consumption $g_t$ such that

$$g_t = \theta_t - \frac{(1-l_t)B_{t-1}}{P_t}, \tag{49}$$

where the dividend $\theta_t$ paid by $M$ is equal to

$$\theta_t = \frac{X_t - RX_{t-1}}{P_t} - b_t^M + \frac{(1-l_t)b_{t-1}^M P_{t-1}}{Q_{t-1}P_t}. \tag{50}$$

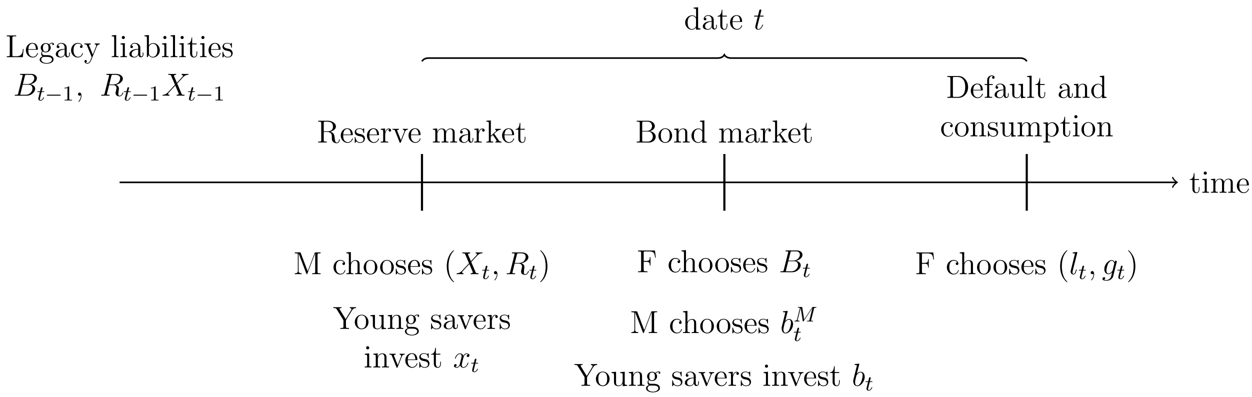


Figure 3: Intradate timing of the game.

Figure 3 summarizes these three stages.

A date-$t$ strategy profile $\sigma_t = (R_t, X_t, x_t, P_t, B_t, b_t^M, b_t, Q_t, l_t)$ describes all the above date-$t$ actions of each agent given all possible history. A strategy profile for the game $\sigma = (\sigma_t)_{t\in\mathbb{N}}$ is the sequence of date-$t$ strategy profiles.

**Objectives of $F$ and $M$.** For all $t \in \mathbb{N}$, the respective date-$t$ objectives of $F$ and $M$ are:

$$U_t^F = \sum_{s\geq t} \beta^{s-t} v(g_s),\ U_t^M = -\sum_{s\geq t} \beta_M^{s-t} \mid P_s - P_s^M \mid, \tag{51}$$

where $\beta, \beta_M \in (0,1)$, there exists $\underline{g} > 0$ such that $v(g) = g$ if $g \geq \underline{g}$ and $v(g) = -\infty$ otherwise, and $P_s^M > 0$ for all $s$.

As in the two-date model, $F$ values spending and is subject to an incompressible level of expenditures $\underline{g}$, whereas $M$ values the price level being on (an exogenously given) target. Unlike in the two-date model, the public authorities incur no exogenous costs of default. We will focus on equilibria in which the private sector's strategy endogenously creates such costs.

**Equilibrium concept.** The equilibrium concept is the same as that in the two-date game—subgame perfection with large and small agents:

**Definition 2.** ***(Equilibrium)*** *An equilibrium is a strategy profile $\sigma$ such that:*

[17]For example, the endowment is constant across cohorts and $r_t \leq 1$.

1. *Each action by F and M is optimal given history and its beliefs that the future actions are taken according to the strategy profile.*

2. *Date-t young saver $i \in [0,1]$ optimally invests $x_t^i = x_t$ in the reserve market given history up to date $t-1$, $(R_t, X_t, x_t, P_t)$, and the strategy profiles for all future actions, and optimally invests $b_t^i = b_t$ in the bond market given history up to date $t-1$, $(R_t, X_t, x_t, P_t, B_t, b_t^M, b_t, Q_t)$, and the strategy profiles for all future actions.*

3. *At each date, the market for reserves clears, $P_t x_t = X_t$, and so does the bond market, $Q_t B_t = P_t(b_t + b_t^M)$.*

This infinite-horizon section focuses exclusively on situations, ruled out by a finite horizon, in which public liabilities are self-sustained Ponzi schemes. Accordingly and for analytical simplicity, we deprive the public sector from any resources other than that generated by such schemes. We abstract in particular from taxation. Our main goal is to show that the important exogenous variables of the baseline model can arise as equilibrium objects of this infinite-horizon setting. More precisely, we endogenize the respective real resources $\bar{x}$ and $\bar{\tau}$ of $M$ and $F$ at date 1 and their respective costs of default $\alpha_M$ and $\alpha_F$ as resulting from their continuation utilities in the infinite-horizon game after dates 0 and 1 have been played.

Consider thus $\bar{x}, \bar{\tau}, \alpha_M, \alpha_F \geq 0$ that satisfy the conditions (8) and (9) of the baseline model. We have:

**Proposition 8.** ***(Endogenous payoffs of the baseline model)*** *If $\beta r_t \leq 1$ for all $t \geq 1$, there exists an equilibrium $\sigma$ such that date 0 is strategically equivalent to date 0 in the baseline model with parameters $\bar{x}, \bar{\tau}, \alpha_M, \alpha_F \geq 0$ and interest rate $r_0$. In other words, the continuation profiles $(\sigma_t)_{t \geq 1}$ generate the same payoffs as that of the baseline model.*

*Proof.* See Appendix E. □

The construction of the equilibrium that endogenizes the exogenous variables of the baseline model, somewhat involved, is detailed in the proof of Proposition 8. Yet the main forces at play are simple: The private sector imposes discipline on the public one by reducing the size of public liabilities in case of default, thereby inducing both reduced public spending and inflation. Dynamic inefficiency is crucial to make such market behavior subgame perfect.

Consider first the fiscal authority. The date-1 default cost $\alpha_F$ imposed by the market to the fiscal authority $F$ is simply a reduction $\alpha_F/\beta$ in the date-2 present value of the Ponzi scheme that the market is willing to sustain on public debt in the event of a date-1 default relative to the case in which $F$ has made good on its date-1 liabilities. The date-1 resources $\bar{\tau}$ are the maximum debt capacity that the market grants to $F$ at date 1. From date 2 on, the private sector discourages default by credibly threatening to stop rolling over debt in case of past credit event. This is effective as the fiscal authority would then be unable to finance its incompressible expenditures.

The cost $\alpha_M$ to the monetary authority $M$ in case of sovereign default is also a form of partial market exclusion, albeit more subtle. In case of default, savers invest only $R_1X_1/(P_2^M + \alpha_M/\beta_M)$ in the date-2 reserve market. This forces a reserve overflow no matter the date-1 monetary policy $(R_1, X_1)$, leading in turn to a date-2 price level off target by $\alpha_M/\beta_M$.

Under this microfoundation of $\bar{x}, \bar{\tau}, \alpha_M, \alpha_F$, $F$'s ability to induce $M$ to inflate away public liabilities is thus driven by the extent to which savers run not only on bonds but also on reserves in the event of sovereign default. The monetary authority is willing to preemptively generate itself the inflation that a run on its currency would generate anyway following a credit event. Thus, in an economy in which the private sector can swiftly switch out of the local currency and "dollarize" in case of a debt crisis (high $\alpha_M$), the monetary authority would be eager to prevent such crises by monetizing sovereign debt even if this comes at a sizeable inflation cost. On the polar opposite, if the private sector has an incompressible demand for reserves whose level is not too far below that of the legacy reserves $R_{-1}X_{-1}$ (low $\alpha_M$), then the central bank can discourage any fiscal attempt at a Sargent-Wallace expansion. It is credible at doing so because there will be no run on its liabilities in the (out-of-equilibrium) event of a sovereign default.

We find it interesting to fully micro-found our baseline model by means of the infinite-horizon one using market-discipline arguments. We offer in particular a simple formalization of the broad idea that a central bank with a pure price-stability mandate may still care about sovereign solvency because default affects the transmission of monetary policy. This is a useful contribution because such an impact of sovereign default on price stability has seldom been modelled to our knowledge. Yet, the study of fiscal and monetary interactions hinges on the assumption that sovereign solvency matters to the monetary authority, albeit often implicitly so as in the pioneering work of Sargent and Wallace (1981).

## 7 Concluding remarks

This paper formalizes Wallace's "game of chicken" as a full-fledged model of strategic dynamic interactions between fiscal and monetary authorities, and investors in their liabilities. We find that a monetary authority that lacks both commitment power and fiscal support may still be in the position of imposing its objectives. Monetary dominance prevails when the implementation of the inflationary fiscal expansion envisioned by Sargent and Wallace (1981) is too costly to the fiscal authority. This may in turn occur because, in the absence of commitment power, inflationary fiscal expansion requires a massive initial debt issuance. The benefits from future inflation may be smaller than the costs from repaying this debt if the interest on it, or/and taxation costs are sufficiently large.

We believe that our framework opens up many avenues for future research on strategic fiscal and monetary interactions, including in particular the three following ones. First, we posit in this first pass that all public liabilities are perfect substitutes. A natural extension is one in which they provide different liquidity services. Second, we restrict the analysis to a perfect-foresight environment, and a study of shocks is in order. Based on our perfect-foresight analysis, we conjecture that the fiscal authority endogenously amplifies shocks above a certain size by doubling down with a Sargent-Wallace expansion when the fiscal situation becomes sufficiently dire. The prudential management of the central bank's balance sheet in anticipation of these amplified shocks is an interesting question. Finally, we focussed on the case in which the agent whose solvency the monetary authority cares about is the government. Yet, we could also consider the case in which such important borrowers belong to the private sector (e.g., financial institutions). The monetary authority would then presumably have to manage a collective moral hazard problem related to that in Farhi and Tirole (2012). The alternative to monetary dominance would in this case be the so-called financial dominance rather than the fiscal one.

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# Appendix

## A Proof of Propositions 3 and 4

If $M$ announces a rate $R$ and issues new reserves $X_0 - R_{-1}X_{-1}$, savers' optimal portfolio choice and market clearing define the date-0 price level $P_0$ and demand for reserves $x$ as the unique solution to

$$R = \frac{rP_1}{P_0} \text{ and } P_0 x = X_0, \tag{52}$$

where $P_1$ is given by the continuation described in Propositions 2 then 1.

Since condition (23) cannot hold if $\beta r \leq 1$, $M$ cannot avoid the Sargent-Wallace debt level in this case. It can announce $R = r(P_1^M + \alpha_M)/P_0^M$ and issue any level of reserves $X_0 - R_{-1}X_{-1} \in [0, P_0^M \bar{x}/r - R_{-1}X_{-1}]$ so that the date-0 price level is $P_0^M$, and the economy unfolds such that $F$ issues the Sargent-Wallace debt level. The date-1 price level is $P_1^M + \alpha_M$ because the upper bound $P_0^M \bar{x}/r$ on $X_0$ rules out a date-1 reserve overflow.

Suppose now that $\beta r > 1$. Using relations (52) to eliminate $R$ and $x$ from condition (23) ensuring that $F$ issues the price-taking debt level yields

$$(\beta r - 1)\left(\bar{x} + \bar{\tau} - \underline{g} - r\left(\underline{g} + \frac{R_{-1}X_{-1} - X_0}{P_0}\right)^+ - \frac{rX_0}{P_0}\right) \geq \frac{\alpha_M r X_0}{P_0(\underline{P}_1 + \alpha_M)}. \tag{53}$$

$M$ can reach $P_0 = P_0^M$ and $P_1 = P_1^M$ by announcing $R = rP_1^M/P_0^M$ and setting $X_0$ below the minimum $X_m$ of two values. First, in order to avoid date-1 reserve overflow, it must be that $X_0 \leq P_0^M \bar{x}/r$, which is compatible with $X_0 \geq R_{-1}X_{-1}$ from assumption (9). Second, $X_0$ must also be smaller than the maximum value such that (53) holds with $P_0 = P_0^M$ and $\underline{P}_1 = P_1^M$. It is easy to check that this is compatible with $X_0 \leq R_{-1}X_{-1}$ if and only if (24) holds. Furthermore, in this case of monetary dominance at each date, reserves $X_0$ are below not only this minimum $X_m$ but also smaller than $P_0^M \underline{g} + R_{-1}X_{-1}$ as $M$'s lexicographic preferences lead it to ensure that $F$ does not consume more than the incompressible minimum at date 0.

If (24) does not hold, monetary dominance at each date is not possible. $M$ can in this case let $F$ issue the Sargent-Wallace debt level and warrant, acting as in the above case $\beta r \leq 1$, that $(P_0, P_1) = (P_0^M, P_1^M + \alpha_M)$, in which case it utility is $-\beta_M \alpha_M$.

Another option is to discourage $F$ from issuing the Sargent-Wallace debt level by manipulating price levels. First $M$ can ensure that $P_1 = P_1^M$ by setting $R = rP_1^M/P_0^*$ and $X_0 = R_{-1}X_{-1}$, where $P_0^*$ is the smallest date-0 price level ensuring that (53) holds with $X_0 = R_{-1}X_{-1}$ and $\underline{P}_1 = P_1^M$. It is easy to see that $P_0^*$ is linearly increasing in $R_{-1}X_{-1}$ and tends to $P_0^M$ as $R_{-1}X_{-1}$ gets close to the largest level warranting monetary dominance. Thus the disutility from this strategy vanishes as $R_{-1}X_{-1}$ tends to this level. Second $M$ may want to manipulate both $P_0$ and $\underline{P}_1$ as the right-hand side of (53) decreases in $\underline{P}_1$. Formally, $P_0$ and $P_1$ solve in this case:

$$\min_{P_0, P_1} P_0 + \beta_M P_1 \tag{54}$$

$$\text{s.t.} \;\; (\beta r - 1)\left(\bar{\tau} - \underline{g} - r\left(\underline{g} - \frac{\bar{x}}{r} + \frac{R_{-1}X_{-1}}{P_0}\right)^+\right) \geq \frac{\alpha_M \bar{x}}{P_1 + \alpha_M}. \tag{55}$$

This strategy cannot dominate that consisting in raising only $P_0$ as $R_{-1}X_{-1}$ tends to the level warranting monetary dominance because it requires issuing a strictly positive quantity of new reserves creating a cost of deterring the Sargent-Wallace debt level that is bounded away from 0.

# B Proof of Proposition 5

Suppose that $M$ announces $R = rP_1^M/P_0^M$ and $X_0 = R_{-1}X_{-1}$, and that savers invest $x$ in the reserve market. We show that for $\alpha_F$ sufficiently large, $F$ chooses the price-level taking strategy in the bond market.

Notice first that for $\alpha_F$ sufficiently large other things being equal, constraint (37) is slack at the solution to (33). Thus in the monetary-dominance strategy, the outcome no longer depends on the value of $\alpha_F$ past a threshold.

An inspection of {(39); (40); (41)} shows that holding $P^F = \underline{P}_1 + \alpha_M$ fixed, $\tau_1$, $b$, and $B$ grow without bounds as so does $\alpha_F$ other things being equal. The properties of the cost of taxation $c$ implies that the utility that $F$ derives from the Sargent-Wallace debt level thus tends to $-\infty$ as $\alpha_F$ grows.

Overall this means that for $\alpha_F$ sufficiently large, $F$ issues the monetary-dominance debt level. This implies in turn that the date-0 reserve market clears at $P_0 = P_0^M$ and

$x = R_{-1}X_{-1}/P_0^M$, and that $P_1 = \underline{P}_1 = P_1^M$ from $\bar{x} \geq rR_{-1}X_{-1}/P_0^M$.

# C Proof of Proposition 6

The only part of the proposition that is not established in the body of the paper is the optimal debt issuance conditional on date-1 fiscal dominance. Suppose that $F$ issues $B$ leading to date-1 fiscal dominance ($P_1 = P^F$).

We first show that $M$ optimally sets $b^M = x - R_{-1}X_{-1}/P_0$ in response to such a $B$ to minimize $P_1 = P^F$. The conditions for bond-market equilibrium (43) together with the definition of $P^F$ (11) yield

$$P^F = \frac{B + RX_0}{\bar{x} + \bar{\tau} + r(1 - b - x)b^M}, \tag{56}$$

and

$$\frac{B}{B + RX_0}(\bar{x} + \bar{\tau}) = br(1 - b - x) + \left(1 - \frac{B}{B + RX_0}\right) b^M r(1 - b - x). \tag{57}$$

Condition (57) implies that given $B$, $r(1 - b - x)b^M$ must increase with $b^M$. Suppose otherwise: Then $b$ must be decreasing as $b^M$ increases. In this case, $r(1 - b - x)b$ is also decreasing in $b^M$. But then the left-hand term of (57) is independent from $b^M$ whereas the right-hand term is decreasing in $b^M$, a contradiction since no equilibrium would form as $b^M$ increases. Condition (56) then implies that $M$ finds it optimal to maximize $b^M$ in order to minimize $P^F$.

Using $b^M = x - R_{-1}X_{-1}/P_0$, one can rewrite (57) as

$$b = \frac{B(\bar{x} + \bar{\tau})}{(B + RX_0)r(1 - b - x)} - \frac{(x - \frac{R_{-1}X_{-1}}{P_0})RX_0}{B + RX_0}, \tag{58}$$

and simple algebra shows that this implies that $b$ increases with respect to $B$. Since $F$ consumes $x - R_{-1}X_{-1}/P_0 + b$, it chooses the maximum $B$ that is compatible with absence of default. That $P^F = RX_0/(\bar{x} + \bar{\tau} - r(1 - b - x)b)$ implies in turn that $P^F$ increases in $B$ (taking into account that $b$ increases in $B$), and so $B$ is such that

$$P_1 = \underline{P}_1 + \alpha_M. \tag{59}$$

# D Proof of Proposition 7

From the previous proof, the real proceeds from the Sargent-wallace debt level $b^{SW}$ solve:

$$b^{SW} = \frac{1}{r(1-x-b^{SW})}\left(\bar{x}+\bar{\tau}-\frac{RX_0}{\underline{P}_1+\alpha_M}\right). \tag{60}$$

As a result, $F$'s utility differential $\Delta$ between the "price-level taking" debt level (such that $P_1 = \underline{P}_1$) and the "Sargent-Wallace" debt level (such that $P_1 = \underline{P}_1 + \alpha_M$) is:

$$\Delta = x - \frac{R_{-1}X_{-1}}{P_0} + b^{PT} + \beta\left(\bar{x}+\bar{\tau}-r(1-x-b^{PT})b^{PT} - \frac{RX_0}{\underline{P}_1}\right) \tag{61}$$

$$- (x - \frac{R_{-1}X_{-1}}{P_0} + b^{SW}) \tag{62}$$

$$= \underbrace{b^{PT}[1-\beta r(1-x-b^{PT})] - b^{SW}(1-\beta r(1-x-b^{SW})}_{A} \tag{63}$$

$$\underbrace{- \beta RX_0\left(\frac{1}{\underline{P}_1} - \frac{1}{\underline{P}_1+\alpha_M}\right)}_{B}. \tag{64}$$

This latter expression of $\Delta$ illustrates the costs and benefits from the price-level taking issuance versus the Sargent-Wallace issuance. Term $A$ measures the difference in utility from allocating consumption over time in different ways across debt levels. The sign of $A$ is ambiguous as the allocation is suboptimal under the Sargent-Wallace issuance but the total to be allocated is larger due to the lower value of reserves. Term $B$ is positive. It is the benefit from eroding the value of reserves $RX_0$ with inflation.

**First stage of date** $0$. Market clearing in the reserve market reads:

$$X_0 = P_0 x, \tag{65}$$

and savers' rationality implies

$$\frac{RP_0}{P_1} = r(1-b-x). \tag{66}$$

Given the continuation of the game derived above, relations (65) and (66) form a system in $(x, P_0)$ as a function of $(R, X_0)$ with a unique solution. We solve for the equilibrium in the two cases covered by Proposition 7: i) $g_1^{PT}(0) > 0$ and $R_{-1}X_{-1}$ sufficiently small; ii) $g_1^{PT}(0) = 0$.

Suppose first that $g_1^{PT}(0) > 0$ and take $R_{-1}X_{-1}$ sufficiently small other things being equal. In this case, $M$ sets $X_0 = R_{-1}X_{-1}$ and announces $R = r(1 - X_0/P_0^M - b^{PT})P_1^M/P_0^M$. ($R_{-1}X_{-1}$ sufficiently small implies that there is no date-1 reserve overflow when $M$ keeps reserves at the minimum level this way.) This corresponds to an equilibrium in which savers invest $X_0/P_0^M$ in the market for reserves and $b^{PT}$ in that for bonds, and the price level is on $M$'s target at each date. The reason is that for $R_{-1}X_{-1}$ sufficiently small, $b^{PT}$ is interior as it converges to $b^{PT}(0)$, and so term $A$ in $\Delta$ is positive, bounded away from 0, whereas the gains $B$ are sufficiently small. In particular, the lexicographic preferences of $M$ imply that minimizing $x$ this way is optimal because this minimizes the distortions in $F$'s choice of $b$ given that prices are on target.

Suppose then that $g_1^{PT}(0) = 0$. In this case, it is always optimal for $F$ to issue the Sargent-Wallace level in the bond market since $A$ is always negative no matter $M$'s actions in the date-0 reserve market: The increase in date-1 resources induced by the lower value of reserves in the Sargent-Wallace debt level relaxes the binding constraint $g_1 \geq 0$ in the consumption-smoothing one. As a result, $\underline{P}_1 + \alpha_M$ is the lowest price that $M$ can hope for at date 1. Since the largest one that it prefers to default is $\underline{P}_1 + \alpha_M$, this has to be the date-1 price. Accordingly, monetary policy in the date-0 reserve market is as follows. Let $y_0$ implicitly defined by

$$y_0 r(1 - y_0) = \bar{x} + \bar{\tau}, \tag{67}$$

and

$$\underline{P}_0 \equiv \max\left\{P_0^M; \frac{R_{-1}X_{-1}r(1-y_0)}{\bar{x}}\right\} \tag{68}$$

$M$ announces a rate $R = r(1-y_0)(P_1^M + \alpha_M)/\underline{P}_0$ and issues $X_0 \in [R_{-1}X_{-1}, \bar{x}\underline{P}_0/r(1-y_0)]$. This sets the date-0 price at $\underline{P}_0$ and $x = X_0/\underline{P}_0$. $M$ in particular may be indifferent across several levels of reserves $X_0$ because any resources that it leaves on the table are borrowed against by $F$ in the bond market, and the utilities of both authorities are unchanged across these levels.

# E Proof of Proposition 8

We prove the proposition in two steps. First, we construct a subset of equilibria indexed by sequences of savings in reserves and bonds. In this subset, equilibria are such that price levels are on target and $F$ does not default. Second, we build an equilibrium that has the properties of the proposition by selecting, from the subset of equilibria that we have constructed in the first step, continuation equilibria contingent on $F$'s date-1 default decision.

**Step 1.** *Let* $(\bar{x}_t, \bar{b}_t)_{t\geq 0}$ *such that* $\bar{x}_0 \geq R_{-1}X_{-1}/P_0^M$, $\bar{b}_0 \geq 0$, $\bar{x}_0 + \bar{b}_0 \geq \underline{g} + R_{-1}X_{-1}/P_0^M$, *and for all* $t \geq 0$*:*

$$\bar{x}_{t+1} = r_t \bar{x}_t,\ \ \bar{b}_{t+1} = r_t \bar{b}_t + \underline{g}. \tag{69}$$

*There exists an equilibrium without default and such that for all* $t \geq 0$, $P_t = P_t^M$, $x_t = \bar{x}_t$, *and* $b_t = \bar{b}_t$.

**Proof.** Define for all $t \geq 0$:

$$P_{t+1}^* = \frac{R_t X_t}{\bar{x}_{t+1}} \tag{70}$$

The strategy profile is the following. At each date $t \geq 0$:

- $M$ announces a rate $R_t = r_t P_{t+1}^M / P_t^M$.
- $M$ issues $X_t = R_{t-1}X_{t-1}$ if $t > 0$ and $X_0 = P_0^M \bar{x}_0$.
- The date-$t$ price level $P_t$ and demand for reserves $x_t$ solve $X_t = P_t x_t$ and $P_t R_t = P_{t+1}^* r_t$ if $R_t > 0$, and $x_t = 0$ otherwise.
- $F$ issues $P_{t+1}^* r_t \bar{b}_t$.
- $M$ does not invest in the bond market ($b_t^M = 0$).
- If $B_t > P_{t+1}^* r_t \bar{b}_t$ then savers shun the bond market ($b_t = 0$). So do they if $t > 0$ and at some $0 \leq t' < t$, $F$ has defaulted ($l_{t'} > 0$). Otherwise the demand $b_t$ and price

$Q_t$ for bonds solve:

$$Q_t B_t = P_t(b_t + b_t^M), \tag{71}$$

$$r_t P^*_{t+1} Q_t = P_t. \tag{72}$$

- $F$ sets $l_t = 0$ as long as this is compatible with $g_t = \theta_t + b_t^M - B_{t-1}/P_t \geq \underline{g}$, where $\theta_t = (X_t - R_{t-1}X_{t-1} + b_{t-1}^M P_{t-1}/Q_{t-1})/P_t - b_t^M$, and defaults otherwise.

We now show that this strategy profile corresponds to an equilibrium with outcome $(x_t, b_t, P_t) = (\bar{x}_t, \bar{b}_t, P_t^M)$ and no default.

Notice first that this strategy profile yields this outcome. First, $X_t = P_t x_t$ and $P_t = P^*_{t+1} r_t / R_t = X_t/\bar{x}_t$ imply $x_t = \bar{x}_t$, and together with $X_0 = P_0^M \bar{x}_0$ this implies in turn that $P_t = P_t^M$. This outcome in the reserve market implies in turn that $b_t = \bar{b}_t$ and that there is no default.

Second, we show that each agent acts optimally given the others' strategies. Savers act optimally given $F$ and $M$'s strategies and market outcomes since they earn $r_t$ on date-$t$ public securities.

Second, given $F$ and the market's strategies, $M$'s strategy is optimal. $M$ reaches its price target at each date. It cannot generate more resources at date $t$ while being on these targets, because the market's strategy in the reserve market implies $x_t = \bar{x}_t$ no matter the values of $R_t > 0$ and $X_t$ from $X_t/x_t = P_t = P^*_{t+1} r_t/R_t = X_t/\bar{x}_t$ as seen above.

Third, $F$'s strategy is optimal given that of $M$ and savers. It dominates any alternative that generates expenditures below $\underline{g}$ at any date or default. On the debt market, $F$ cannot issue more than $P^*_{t+1} r_t \bar{b}_t$ as savers would credibly shun the bond market forever in this case. Thus the highest possible real ressource extracted on the debt market is $b_t = \bar{b}_t$ due to the date-t market's strategy and future strategies.

**Step 2.** We now construct an equilibrium that has the properties claimed in the Proposition. First, strategies from date 2 on depend on whether there has been default at date 1 ($B_0$ and $l_1$ strictly positive) or not.

In the absence of date-1 default, the date-2 continuation equilibrium is as in Step 1 taking date 2 as the initial date with $\bar{x}_2^{ND} = \bar{x} r_1$ and $\bar{b}_2 = \bar{b}_2^{ND}$ taken above a lower bound specified below. The only difference is that we add the condition that date-$t$ savers shun the date-$t$ bond market if $F$ raised more than $\bar{\tau}$ at date 1. This pins down the date-1 debt capacity of $F$ at $\bar{\tau}$.

In case of date-1 default, then the date-2 continuation equilibrium is such that $\bar{x}_2^D = R_1X_1/(P_2^M + \alpha_M/\beta_M)$, implying that $P_2$ cannot be smaller than and is in equilibrium equal to $P_2^M + \alpha_M/\beta_M$. Accordingly, $M$ announces a rate $r_2P_3^M/(P_2^M + \alpha_M/\beta_M)$ at date 2. Furthermore $\bar{b}_2^D = \bar{b}_2^{ND} - \alpha_F/\beta - r_1B_1(1/P_2^M - 1/[P_2^M + \alpha_M/\beta_M)] \geq \underline{g} + r_1\bar{\tau}$, and this latter inequality puts a lower bound on $\bar{b}_2^{ND}$. Finally, we add again the condition that date-$t$ savers shun the bond market if $F$ raised more than $\bar{\tau}$ at date 1.

This profile from date 2 on implies that $F$ always raises exactly $\bar{\tau}$ in the date-1 bond market, and faces a cost of default in the form of a loss in date-2 resources whose date-1 present value is $\alpha_F$. $M$ faces a date-2 run on its reserves in case of date-1 default, with a cost $\alpha_M$ viewed from date 1. Overall, $F$, $M$, and savers face the same date-1 payoffs viewed from date 0 as in the baseline model.