Shadow Banking and Bank Capital Regulation

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Abstract

Banks are subject to capital requirements because their privately optimal leverage is higher than the socially optimal one. This is in turn because banks fail to internalize all costs that their insolvency creates for agents who use their money-like liabilities to settle transactions. If banks can bypass capital regulation in an opaque shadow banking sector, it may be optimal to relax capital requirements so that liquidity dries up in the shadow banking sector. Tightening capital requirements may spur a surge in shadow banking activity that leads to an overall larger risk on the money-like liabilities of the formal and shadow banking institutions. (JEL G01, G21, G28)

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The U.S. banking system now features two components of equal importance, traditional banks and the so-called “shadow banking system.” This term refers to the nexus of financial institutions that perform the function of traditional banks by financing loans with the issuance of money-like liabilities while not being subject to the prudential regulation of banks. Pozsar et al. (2010) offer an excellent detailed description of the shadow banking system, and show that its total liabilities have the same order of magnitude as that of the traditional banking sector.

Most observers agree that the growth of shadow banking has been largely driven by regulatory arbitrage (see, e.g., Acharya, Schnabl, and Suarez 2013; Gorton and Metrick 2010; Pozsar et al. 2010). Traditional banks are subject to regulatory capital requirements that put an upper bound on their leverage. The shadow banking system enabled the refinancing of bank assets with near-monies, such as money market funds shares with a higher leverage than that imposed on traditional banks. As a result, the growth of the shadow banking system has greatly increased the effective leverage on loans to the U.S. economy and, in particular, the amount of money-like liabilities backed by these loans.

In the face of the costs that the 2008 banking crisis has created for the world economy, a global trend towards imposing heightened capital requirements on traditional banks has emerged. On the other hand, regulatory reforms remain thus far largely silent on many aspects of shadow banking (Adrian and Ashcraft 2012; Gorton and Metrick 2010). As noted by several observers (e.g., Adrian and Ashcraft 2012; Kashyap, Stein, and Hanson 2010; Stein 2010), this raises the possibility that heightened capital requirements for traditional banks trigger even more regulatory arbitrage than was observed in the recent past, thereby inducing a large migration of banking
activities towards a more lightly regulated shadow banking system. The higher solvency of the traditional banking system then may be more than offset by such growth in shadow banking. This may ultimately lead to a higher aggregate exposure of the money-like liabilities of the formal and shadow banking sectors (deposits, money market funds shares, and repurchase agreements) to shocks on loans.

Implicit in this view is the assumption that a rapid pace of financial, legal, and accounting innovations enables financial institutions to create and exploit regulatory arbitrage opportunities, thereby relaxing the economic constraints that prudential rules aim to subject them too. The goal of this paper is to develop a framework to study optimal bank capital regulation in the presence of such regulatory arbitrage.

It has been a long-standing idea that the rise of new forms of financial contracts is often triggered by regulatory constraints (see, e.g., Silber 1983; Miller 1986; Kane 1988). The arbitrage of capital requirements has become in particular an important feature of the banking industry since the implementation of the first Basel accords. Yet, formal models of optimal bank capital regulation with such imperfect enforcement are lacking. Given the current regulatory agenda, it seems important to develop frameworks for the analysis of prudential regulation in which the possibility of regulatory arbitrage is taken seriously. This paper studies the optimal prudential regulation of banks in the presence of a shadow banking sector.

1“The dilemma of the current regulatory reform efforts is that the motivation for shadow banking has likely become even stronger as the gap between capital and liquidity requirements on traditional institutions and non-regulated institutions has increased.” (Adrian and Ashcraft 2012) “(. . .) the danger is that, in the face of higher capital requirements, these same forces of competition are likely to drive a greater volume of traditional banking activity into the so-called ‘shadow banking’ sector.” (Kashyap, Stein, and Hanson 2010)

2Kashyap, Stein, and Hanson (2010) survey the evidence.
The paper proceeds in two steps. It first presents a baseline model of optimal bank capital regulation with perfect enforcement. It then studies a modification of this model in which banks can bypass prudential regulation using the shadow banking sector.

In the baseline model, banks issue liabilities backed by their loan portfolios to nonfinancial agents. Banks use the proceeds to fund their portfolios, and to pay dividends to their shareholders, who in turn invest them in profitable investments outside the banking industry. Nonfinancial agents use their claims to banks to settle mutually beneficial transactions. Crucially, the risk on bank liabilities undermines this role as a medium of exchange, and destroys gains from trade among nonfinancial agents. A given bank does not fully internalize this cost when issuing liabilities because surplus from trade is split between its customers and their trading partners. As a result, the privately optimal bank leverage is higher than the socially optimal one. There is room for a prudential regulation that caps the risk-adjusted leverage of banks, and that is binding in equilibrium. An important feature of this regulation is that banks commit to a leverage ratio that is not contingent on the private information that they may receive about their lending opportunities. This reduces adverse selection and maximizes the cash that they raise per dollar of liabilities issued. This corresponds to the fact that banks typically back their deposits with a large pool of very diverse asset classes in a single balance sheet.

I then introduce shadow banking as follows. I assume that bankers can bypass capital requirements because the regulator cannot observe their transactions with money market funds (MMFs) that also issue money-like liabilities to nonfinancial agents. Banks can use the shadow banking sector to pledge a larger fraction of their portfolios than prudential regulation permits.
in principle. Crucially, banks and MMFs interact in a spot market in which banks cannot commit to not use their private information. Thus, they face adverse selection costs: their portfolios are less liquid in the shadow banking sector than in the formal one. The upside from this adverse selection is that it induces banks to raise less cash in the shadow banking sector than they would absent this friction. The downside is that they issue larger liabilities and thus transfer more risk per dollar raised.

There are two locally optimal regulatory responses to such regulatory arbitrage. First, the regulator can tighten capital requirements, triggering an increase in shadow banking activity and making banks that are not willing to incur adverse selection costs very safe. Second, the regulator may also prefer to relax regulatory capital requirements so as to bring shadow banking activity back in the spotlight of regulation. Current regulatory reforms seem to trend towards the former solution. The latter one may yet be preferable, particularly so if informational frictions in the shadow banking sector are not important.

Although the paper focuses on the impact of tighter capital requirements on shadow banking activity, the model has implications beyond this specific question. It makes the more general point that regulating the riskiness of the liabilities issued by financial institutions may be counterproductive when the regulator cannot measure risk as accurately as can the other agents. A tighter regulation then results in financial institutions tending to transfer riskier claims to their creditors, and this endogenous response may more than offset the direct impact of regulation.

This paper relates to two strands of literature. First, it offers a simple (yet fully microfounded) model for the role of bank deposits as media of exchange and the resulting externalities from bank insolvencies. Whereas
the traditional banking literature has mostly emphasized the role of banks in maturity transformation, the key benefit of banks in this paper is that they provide nonfinancial agents with claims that they can use to settle trades. The rationale for prudential regulation is then that banks do not capture the full surplus from trade, and hence do not internalize the full social cost of bank failure. Gu et al. (2013) explain this role of bank liabilities as near monies with the superior commitment power of banks. I take this role for granted, and study its implications in terms of socially and privately optimal bank balance sheets. Kahn and Santos (2010) also assume such a role for bank liabilities in facilitating trade, and show that banks may fail to fully internalize the social costs of their exposures to each other’s risk in the interbank market in this case.

This paper also relates to the literature on the interaction between banks and markets. Bolton, Santos, and Scheinkman (2011) study the ex ante impact on banks of ex post adverse selection in the secondary market for their assets. In their setup, the fear of future adverse selection may induce banks to offload their risky assets too early so as to sell them at fair value. This is inefficient because this implies that the suppliers of liquidity to banks hoard less cash to snap up these assets given a lower expected return on them. This in turn reduces the total quantity of valuable risky assets in which banks decide to invest in the first place. By contrast, I emphasize that adverse selection in secondary markets for bank assets may be ex ante desirable as it reduces the scope for regulatory arbitrage.

Gennaioli, Shleifer, and Vishny (2013) develop a model of shadow banking in which banks pool their idiosyncratic risks, thereby increasing their systematic exposure, and use the safe part of these recombined portfolios to back the issuance of safe debt. Although this is efficient under rational ex-
pectations, shadow banking creates large financial instability and systemic risk when agents underestimate the tail of systematic risk.

Ordonez (2013) develops an interesting model of shadow banking in which unregulated banking can be superior to regulated banking if (1) regulation inefficiently restricts risk taking by banks and (2) reputational concerns are an effective disciplining device in the shadow banking sector. This relates to the broad point also made here that if regulation is inefficient, then a shadow banking sector might be desirable, although the rationale is quite different.

Finally, Harris, Opp, and Opp (2014) develop a model in which capital requirements for banks may be counterproductive like in this paper, but for a very different reason. In their model, capital requirements reduce the funding capacity of banks. This spurs entry by nonbanks in the business of lending to good borrowers. This induces banks to focus on lending to bad borrowers for which their profits are generated by the government put, rather than by the intrinsic value of the projects that they fund.

1. Unregulated Banking

There are two dates: $t = 1, 2$. There are four agents: a household, an entrepreneur, the shareholder of a bank, and the manager of a money market fund, who is simply referred to as the "MMF" henceforth. All agents are risk neutral over consumption at dates 1 and 2, and cannot consume negatively. They do not discount future consumption. There is a consumption good that is valued by all agents and used as the numéraire.

The household receives a date-1 endowment of $W$ units of the numéraire good, where $W > 0$. 
The entrepreneur has access to a technology that enables him to produce a second consumption good. Only the household derives utility from consuming this second good. Production takes place at date 2. However, the entrepreneur must decide at date 1 on a production scale $N_1$ - a number of units to be produced at date 2. He can revise this production scale at date 2 from $N_1$ to $N_2$ before producing, but this comes at an adjustment cost

$$\frac{k}{2} (N_2 - N_1)^2,$$

where $k > 0$. Once the scale is fixed, the production of each unit of output comes at a disutility $c$ to the entrepreneur, where $c \in (0, 1)$. The household values one unit of output as much as one of the numéraire good.

The entrepreneur cannot commit to not withdraw his human capital at date 2. He can always walk away at no cost, thereby not producing any output. He can use this threat to renegotiate any arrangement, and has all the bargaining power during such renegotiations.

Because $c < 1$, there are potential gains from trade between the household and the entrepreneur, but they cannot reap them because of this commitment problem. The entrepreneur would never make good on a promise to deliver output at date 2 against a date-1 payment by the household. The initial household endowment $W$ therefore must be stored from date 0 to date 1, so that the household and the entrepreneur can trade numéraire for the entrepreneur’s output at date 1. Both the bank and the MMF supply such stores of value to the household.

The manager of the money market fund is penniless. He is endowed with a storage technology that is linear with unit return.

The bank shareholder is penniless, and has access to two investment op-
opportunities. First, he can originate a loan portfolio, which is simply modeled as an investment opportunity that requires an initial outlay \( I > 0 \) at date 1 against a repayment at date 2. If the portfolio is of high quality ("good"), it repays \( L + l \), while it repays \( L \) if of low quality ("bad"), where

\[
0 < L < L + l.
\]

The quality of the portfolio is not observable. All agents share the common prior belief that the portfolio is of high quality with probability \( p \in (0, 1) \). The outlay \( I \) must be funded using the household’s resources, and the bank can pledge all or part of the portfolio’s expected payoff to the household.

Second, the shareholder of the bank also has access to an alternative investment opportunity at date 1. If he invests \( x \) units in this opportunity, this generates a gross return \( x + f(x) \) with probability \( q \), and \( x \) only with probability \( 1 - q \). The function \( f \) satisfies the Inada conditions. I assume that this gross return cannot be pledged to the other agents, so that any investment in this alternative technology must be entirely financed by the bank shareholder. Following Bolton, Santos, and Scheinkman (2011), non-pledgeability could stem, for example, from the fact that this is a long-term investment that pays off only at some remote date 3, at which only the bank shareholder values consumption.\(^3\) This alternative opportunity stands for, for example, equity investments outside the banking industry. The quality of the loan portfolio and the return on the alternative investment are independent random variables.

The exact timing of the interaction between these agents is the following:

\(^3\)This assumption could be weakened: I could allow for partial pledgeability of the cash flows generated by this investment opportunity, only at the cost of adding some complexity.
• at date 1, the household stores its resources with the bank and the MMF. The bank issues a security backed by the portfolio and the household bids competitively for it. The entrepreneur makes an initial capacity choice, and

• at date 2, the household receives the proceeds from storage. The entrepreneur, observing this, makes a capacity-adjustment choice and makes a take-it-or-leave-it trading offer to the household.

The analysis focuses on the subgame perfect equilibria of this economy. Finally, the following parameter restrictions simplify the analysis:

1.1 Equilibrium

I solve backwards for the equilibria in this economy. Consider first date 2. Suppose that the household has an endowment $W_2$ at date 2 and that the entrepreneur has chosen a capacity $N_1$ at date 1. The following lemma characterizes their date-2 interaction.

**Lemma 1.** The entrepreneur always sets a maximal output price of one per unit and adjusts his capacity from $N_1$ to $N_2 = W_2$. His profit is therefore

\[
(1 - c)W_2 - \frac{k(W_2 - N_1)^2}{2}
\]

Their respective roles will be explained in due course.
Proof. See the Appendix. ■

Given his commitment problem, the entrepreneur behaves as a monopolist that maximizes his ex post profits after uncertainty resolves at date 2. Condition (2) implies sufficiently low capacity-adjustment costs so that the entrepreneur finds it optimal to charge the household his willingness to pay of one per unit, and to adjust capacity so as to match the household’s resources at this price. This ex post interaction implies that the entrepreneur chooses his initial capacity $N_1$ as follows.

**Lemma 2.** If the entrepreneur believes that the date-2 resources of the household are a random variable $\tilde{W}_2$, he chooses an initial capacity

$$N_1 = E(\tilde{W}_2),$$

and has an expected utility

$$U_E = (1 - c)E(\tilde{W}_2) - \frac{k}{2}Var(\tilde{W}_2).$$

(4)

Proof. See the Appendix. ■

Adjustment costs imply that the entrepreneur would like to eliminate uncertainty on date-2 demand. If the entrepreneur could commit to a contract, this could be achieved by efficient contracting. The entrepreneur and the household could enter into a private deposit-insurance scheme that saves adjustment costs. The household could agree to pay more than one for the output when its date-2 income is large, against the promise that the entrepreneur charges a price smaller than one for the same quantity when its income is low. The entrepreneur’s inability to commit implies that he maximizes ex post profits, which precludes such private insurance mechanisms.
Thus, this contracting friction implies that the riskiness of the storages offered by the financial system to nonfinancial agents has a real impact. The assumption of symmetric quadratic adjustment costs is only for tractability because it implies that variance is the relevant risk measure. One could introduce more realistic, but less tractable, costs that arise only in case of large downsizings.

Consider now the date-1 interaction between the household and the suppliers of stores of value, i.e., the MMF and the bank. From Lemma 1, if the household has resources $W_2$ at date 2, the entrepreneur keeps him at his date-2 reservation utility $W_2$. The household is therefore willing to store any amount between dates 1 and 2 rather than consume it at date 1, as long as the expected gross rate of return is larger than one. Because the MMF can offer a return on storage of at most one to the household, he has no choice but to offer exactly one. This implies that the household prices the securities issued by the bank so that it earns a unit expected return.

The equilibrium is thus entirely characterized by the portfolio-backed security that the bank designs and pledges to the household. Such a security is in turn characterized by its payoff when the portfolio is good and that when it is bad. A generic security is fully described with two numbers $\lambda, \mu \in [0, 1]$, such that the household receives $\mu L$ if the portfolio turns out to be bad and $\mu L + \lambda l$ when it is good, against a date-1 cash payment of $\mu L + \lambda pl$. It must be that

$$\mu L + \lambda pl \geq I;$$

otherwise, the bank cannot fund its loan portfolio. The net cash $\mu L + \lambda pl - I$ is paid out as a dividend to the bank shareholders, who reinvest it in their redeployment opportunity outside the bank. The household stores its
residual wealth \( W - \mu L - \lambda pL \) with the MMF.\(^4\) For such a contract \((\lambda, \mu)\), the date-1 endowment of the household is distributed as

\[
\begin{cases}
W + (1 - p) \lambda l \text{ with prob. } p \\
W - p \lambda l \text{ with prob. } 1 - p.
\end{cases}
\]

From Lemma 2, the expected utility of the entrepreneur for such a contract is therefore

\[
U_E = W(1 - c) - \frac{k}{2} p(1 - p) \lambda^2 l^2, \tag{5}
\]

whereas that of the bank shareholder is:

\[
\begin{align*}
U_B &= \mu L + \lambda pl + (1 - \mu)L + p(1 - \lambda)l - I + qf(\mu L + \lambda pL - I) \\
&= pl + L - I + qf(\mu L + \lambda pl - I). \tag{6}
\end{align*}
\]

The bank shareholder receives \( \mu L + \lambda pl \) from the household at date 1, and expects a payoff of \((1 - \mu)L + p(1 - \lambda)l\) on his residual stake.

It is transparent from (6) that the bank utility increases w.r.t. \( \lambda \) and \( \mu \). The bank values cash at date 1 more than at date 2 because of its date-1 nonpledgeable investment, and thus prefers to issue the largest possible security against the portfolio. Absent any regulatory constraint, it always chooses \( \mu = \lambda = 1 \), thereby creating the largest possible adjustment costs for the entrepreneur. Of course, only the risky part of the portfolio payoff creates negative externalities for the entrepreneur. Accordingly, his utility does not depend on \( \mu \).

**Lemma 3.** The unregulated bank chooses to pledge its entire portfolio: \( \mu = \lambda = 1 \).

\(^4\) I assume that \( W \) is sufficiently large that this term is always positive.
Proof. See discussion above.

This way, the model captures in an elementary fashion the idea that banks’ optimal private leverage is excessive because banks do not internalize all the costs induced by the riskiness of their liabilities. This is in turn because these liabilities are near-monies. They serve to facilitate transactions between a bank’s customer and another agent (the entrepreneur) who is not a customer of this bank. Thus, the bank does not fully internalize any reduction in surplus from such transactions due to the riskiness of its liabilities.

The model of leverage externalities presented here is kept very simple because it is only one of the ingredients of the analysis. Yet, it is based on first principles, and could be extended along several dimensions in future work and for other applications. In particular, there are several ways to ensure that the bank shareholder partially internalizes the costs to the real economy caused by the riskiness of his liabilities, so that his capital-structure problem has an interior solution.

Suppose, for example, that the household extracts a fraction of the surplus from trade with the entrepreneur (e.g., the household can make an offer to the entrepreneur with some probability) and that the MMF is competitive. In this case, the bank would face a convex cost of debt, because it would have to compensate the household for making a risky deposit—thus raising expected adjustment costs and in turn lowering expected surplus—rather than investing in the MMF. More generally, one could write a more symmetric model in which nonfinancial agents make deposits first, then search for trading partners and incur real costs if the value of their deposits is uncertain when they withdraw them to execute trades with a partner. Clearly, there would still be leverage externalities as long as all the agents
do not make deposits at the same monopolistic bank. Each bank would not internalize the entire fraction of the surplus that its leverage decision destroys.

2. The Prudential Regulation of Banks

Suppose now that there is an initial date \( t = 0 \) at which a regulator grants a banking license to the shareholder of the bank. A banking license is the exclusive right to originate the loan portfolio described above at date 1. The regulator is entitled to regulate as he sees fit the design of the securities that the bank issues at date 1. He perfectly observes the securities that the bank sells to the household and possibly to the MMF at date 1, so that the bank and the household cannot bypass regulation through the MMF. The goal of the regulator is to maximize the utilitarian welfare of the other agents as expected from date 0. At date 0, the regulator announces the security design that the shareholder of the bank must comply with if he were to accept the license. Both parties can fully commit to this design.

Condition (1) implies that the regulator finds it optimal to grant a banking license: this inequality states that the surplus from originating the loan, even excluding possible gains from investments in the alternative opportunity, is larger than the maximal adjustment costs for the entrepreneur. The security design that the regulator can impose on the bank shareholder at date 0 is again characterized by two numbers \( \lambda, \mu \in [0, 1] \) such that the bank pledges \( \mu L \) to the household if the portfolio is bad and \( \mu L + \lambda l \) if it is good. It must be that

\[
\mu L + \lambda pl \geq I;
\]
on otherwise, the bank cannot fund the loan portfolio. For such a security,
expression (5) and (6) imply that utilitarian welfare is

$$L + pl - I + qf(\mu L + \lambda pl - I) + W(1 - c) - \frac{k}{2}p(1 - p)\lambda^2 l^2.$$  

(7)

Maximizing it is a simple convex problem that yields:

**Proposition 1.** The optimal security \((\lambda^*, \mu^*)\) is such that \(\mu^* = 1\), and \(\lambda^* \in (0, 1)\) is the unique solution to

$$q f' (\lambda^* pl + L - I) = \lambda^* k (1 - p) l.$$  

(8)

It admits a simple implementation, whereby the regulator imposes only a capital requirement on the bank. The regulator imposes that the bank retains a minimum equity stake in the portfolio and issues (risky) debt. The fraction of the portfolio payoff that backs debt cannot exceed

$$\frac{\lambda^* pl + L}{pl + L}$$

**Proof.** See the Appendix.

The fraction \(\lambda^*\) is the central endogenous parameter of the model. It admits a natural interpretation as the net bank leverage or the risk-adjusted bank leverage. After the household has paid an amount \(\lambda^* pl + L\), which is used to fund the portfolio, and to pay a dividend \(\lambda^* pl + L - I\) to the shareholder, the bank has indeed the following balance sheet:

Figure 1 here.

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\(^5\)Clearly, the MMF and the household are always kept at their reservation utility and thus can be ignored.
The asset side features the portfolio. The liability side is comprised of two claims, with that of the household having absolute seniority over that of the bank shareholder (hence the term "shareholder" that anticipated the optimal design). The household’s liability has a perfectly safe tranche $L$ backed by a deterministic cash flow, and a riskier tranche that pays off only if the portfolio is of high quality. In other words, debt net of cash-like assets represents a fraction $\lambda^*$ of the risky part of the assets. In this sense, $\lambda^*$ is the net leverage. Such a risk-adjusted leverage is typically the variable on which current prudential regulations set an upper bound.

It is transparent from (8) that the regulatory net leverage $\lambda^*$ is larger when the opportunity cost of bank capital is high ($q$ large), and the negative externalities that the bank imposes on the real economy are small ($k$ small). It is also easy to see that an increase in $p$ holding $pl$ fixed leads to a higher regulatory leverage. That is, capital requirements should be risk based, and should put more weight on assets with a riskier cash-flow distribution (in the sense of second-order stochastic dominance).

This model thus captures parsimoniously the broad terms of the debate on bank capital requirements. On one hand, heightened capital requirements reduce the negative externalities that bank failures impose on the rest of the economy by undermining the role of bank claims as reliable stores of value. On the other hand, allocating more bank equity to the financing of loans comes at a cost if there is no perfect substitute to this equity capital for the financing of alternative valuable projects.

This stripped-down model can be extended in several directions. The Online Appendix offers an extension in which the payoff of the loan portfolio is continuous. I show that it is still the case that standard debt, that is, a senior tranche on the portfolio payoff, is the optimal design for the house-
hold’s claim. The Online Appendix also discusses the interplay of capital requirements and (imperfect) deposit insurance in this framework. Another important extension that I leave for future research consists of taking into account the general equilibrium effects of bank regulation. One possible effect could stem, for example, from a feedback loop between the risk borne by entrepreneurs and the riskiness of loan portfolios in a closed model in which banks lend to entrepreneurs.

3. Prudential Regulation with Information Problematic Assets

I now add informational frictions to this model of bank regulation. Suppose that the bank shareholder acquires two pieces of private information at date 1. He privately observes the quality of the loan portfolio, and whether or not the alternative investment generates the excess return $f$. Thus, there are four bank “types” at date 1: the bank may have access to a good or a bad portfolio, and may or may not have a valuable alternative investment. The rest of the model is unchanged. In particular, all agents share the date-0 prior belief that the portfolio is good with probability $p$, and that the alternative investment is valuable with probability $q$.

The assumption that the bank privately observes the quality of the loan portfolio simply captures the longstanding view in banking economics that banks have a superior ability to acquire soft information for screening potential borrowers. The assumption that the bank shareholder privately observes the characteristics of his alternative option is a convenient way to parametrize the magnitude of the adverse-selection problem caused by his inside information about the portfolio. The optimal security is the same as
that absent private information:

**Proposition 2.** The optimal security does not depend on the date-1 type of the bank. It is identical to that described in Proposition 1: the bank commits to pledge the safe part of the portfolio payoff and a fraction $\lambda^*$ of its risky part that solves (8).

**Proof.** See the Appendix.

There is pooling across date-1 bank types. Regardless of its date-1 private information, the bank again issues risky debt against its portfolio, and retains a level of equity that depends only on the initial public information available about the portfolio. The only difference with the symmetric information case is that the leverage $\lambda^*$ is no longer an upper bound, but rather a target that the bank must reach at date 1. Unlike in the symmetric-information case, long-term contracting is important because such pooling is not ex post optimal for the bank. If it was making an ex post optimal financing decision, a bank that has a good portfolio and no valuable alternative investment would not raise more than $I$ from the household at date 1. Pooling is ex ante optimal, however. The intuition for this is simple. The goal of the regulator is that a bank with a valuable alternative opportunity raises as much cash as possible at date 1 for a given level of risk transferred to the household. Given that a bank with a bad portfolio always seeks to mimic the type that raises as much cash as possible at date 1, the regulator achieves this by asking that a bank with a good portfolio and no valuable alternative opportunity pools with a bank that has a good portfolio and an opportunity. This minimizes the cost of date-1 adverse selection for this latter bank type, and this does not come at any ex ante cost to the bank because it is just a subsidy from one of its future types to the others.
Again, this optimal mechanism admits a very simple and realistic implementation. The regulator only needs to impose an upper bound on the risk of bank’s liabilities viewed from date 0, measured as the variance transferred to the household. Actual prudential regulations typically involve value-at-risk computations that aim at a similar goal. Then, subject to this risk-transfer constraint, the bank finds it privately optimal to commit to a pooling behavior whereby it aims at the same leverage regardless of its realized type. The reason is again that this maximizes the amount of cash raised at date 1 subject to the risk-transfer constraint. Banks achieve such pooling in practice by warehousing very diverse classes of loans in a single balance sheet that also receives their deposits, instead of creating many licensed subsidiaries with many different capital levels.\footnote{Leland and Pyle (1977) or Diamond (1984) offer alternative, yet related, rationales for such pooling by banks.} This pooling is in sharp contrast with the way banks refinance assets in securitization markets, or more generally transfer risks in the shadow banking sector. They typically have a greater discretion at picking the assets that they pledge in such markets, at least at the asset-class level. I now introduce such alternative refinancings in the model.

4. Shadow Banking

This section introduces shadow banking in the model of the previous section, and studies how this affects prudential regulation. The following subsection first describes the innovations and prudential and accounting loopholes that enabled the precrisis growth and transformation of the shadow banking sector.
4.1 Shadow banking as unlicensed banking\textsuperscript{7}

As already mentioned, the rise of the shadow banking sector has been largely motivated by regulatory arbitrage. Shadow banking aimed at recreating outside the regulatory umbrella the basics of banking: backing near-monies with risky long-term loans. In the shadow banking sector, money market funds shares played the role of deposits, while asset-backed securities (ABS) held by asset backed commercial paper (ABCP) conduits played that of long-term assets. MMFs shares are perceived as quasi-deposits because MMFs are allowed to subscribe and redeem their shares at a fixed net asset value (typically set to $1). MMFs are allowed to do so in turn because they are managed in accordance with Rule 2a-7 under the Investment Company Act of 1940, which puts restrictions on the maturity, quality, and dispersion of their investments. Although MMFs have traditionally invested in assets that were ultimately backed by very safe short-term claims such as receivables, their exposure to riskier bank loans increased dramatically between 1990 and the outburst of the crisis. This was driven by a sharp increase in the share of ABCP in their investments (from 5.7% to 57% between 1990 and 2007), and by a large shift of ABCP conduits from short-term investments to investments in ABS backed by loans.

This evolution of MMFs' assets begs two questions. First, why did MMF shares continue to be perceived as very safe despite such massive asset substitution? The reason is that the banks sponsoring the ABCP conduits were granting guarantees to their conduits, mostly explicitly but also implicitly for reputational reasons. Second, why did the sponsoring banks find it economical to refinance loans with such guaranteed ABCP conduits? The

\textsuperscript{7}This subsection summarizes, and therefore borrows without restraints from, detailed accounts by Acharya, Schnabl, and Suarez (2013) and Kacperczyk and Schnabl (2010).
answer is that the capital requirements for such guarantees were much lower than those that would have prevailed, had the sponsoring banks held the assets in their balance sheet instead of transferring them to conduits that they then guaranteed.

This difference in prudential treatments of economically similar situations followed from two main accounting and prudential loopholes. First, the guarantees granted by sponsors to conduits were nominally liquidity guarantees rather than credit guarantees. Sponsoring banks were supposed to buy back loans at face value upon the conduit’s request only in the event that no credit losses had occurred, where credit losses were formally defined as significant changes in variables such as delinquency rates. This justified low capital charges. In practice, however, it becomes evident that a portfolio is not performing long before credit losses defined this way materialize. Hence, these liquidity guarantees acted in fact as credit risk transfers from the conduit to the sponsor. Second, in 2004, the U.S. regulator permanently exempted banks from consolidating such liquidity-supported conduits. This way, banks were originating and bearing credit risk without incurring the related capital requirements.

Beyond the specifics of the 2008 crisis, regulatory arbitrage by banks typically amounts more broadly to finding alternative legal and accounting classifications for transactions that would be privately uneconomical given regulation under the standard classification, so that they can be carried out outside the scope of regulation. For example, having MMFs with a fixed net asset value investing in ABCP backed by ABS is economically close to commercial banking, but legally, and therefore prudentially, quite different. In practice, these regulatory arbitrages exploit fine details and subtle loopholes in accounting rules and prudential regulations. These details vary over time,
but the principle remains. I now introduce this view of regulatory arbitrage in the model.

4.2 Shadow banking and bank capital

In line with the above discussion, I introduce shadow banking in the model of Section 3 as follows. I now suppose that while the regulator observes the date-1 trade between the bank and the household, the bank and the MMF can trade secretly at date 1. The rest of the model is unchanged. Thus, while the bank and the regulator can agree at date 0 on a direct risk transfer from the bank to the household and commit to it, the regulator cannot prevent the bank from offloading more risk on the household indirectly through the MMF, who can invest the funds that he collects from the household with the bank instead of using his linear technology. I deem this unregulated interaction the shadow banking sector.

The sale of a stake in the portfolio to the MMF outside the regulatory umbrella may be interpreted as transferring this stake to an off-balance sheet vehicle that is ultimately refinanced by money market funds. This is in line with the evolution already detailed, whereby money market funds shifted investments from standard commercial paper backed by short-term claims to assets backed by bank loans. In practice, risk was not directly transferred to MMF shareholders as I model it. Explicit or implicit bank guarantees implied that this risk was ultimately borne by banks, even though accounting and prudential rules did not reflect this. I abstract from this extra layer of institutional details here: it is immaterial in my model whether the household gets direct exposure to the risk of the portfolio through his holding of MMF shares, or indirectly through the risk that the bank guarantee creates for its deposits. All that matters is the possibility for the bank to pledge
stakes in its loan portfolio outside the regulator’s supervision.

The key ingredient of the model is that the bank trades ex post optimally with the MMF in a spot-market interaction at date 1. This inability to trade in an ex ante optimal fashion is the only distinction between formal and shadow banking. I view this as a parsimonious and natural way to distinguish between the arm’s length interactions in the opaque over-the-counter markets of the shadow banking sector and the long-term, more open-book, relationship between banks and their supervisors.

I now solve for the optimal security chosen by the regulator in the presence of this additional friction. Clearly, the additional friction of secret trading between the bank and the MMF leaves two aspects of the optimal security in Proposition 2 unchanged. First, it is still optimal that the bank sells the entire risk-free cash flow \( L \) to the household. It is in fact immaterial whether the sale of this risk-free cash flow takes place under the regulatory umbrella or in the shadow banking sector. Second, it is still optimal that, as in Proposition 2, the regulator asks all date-1 bank types to pool and sell the same fraction \( \lambda \) of the risky cash flow \( l \). This maximizes the cash raised by the bank under the regulatory umbrella for a given level of risk transfer.

Determining the optimal contract enforced by the regulator thus merely consists in solving for this optimal regulatory leverage \( \lambda \) in the presence of shadow banking. I now solve for such an optimal regulatory leverage in two steps. I first fix an arbitrary regulatory leverage \( \lambda \), and characterize the resulting activity in the shadow banking sector. In a second step, I determine the optimal regulatory leverage when the regulator rationally anticipates such shadow banking activity induced by capital regulation.

4.2.1 Shadow banking activity for a given regulatory leverage.
Suppose that the net regulatory leverage is \( \lambda \in (0, 1) \). The bank may then
sell all or part of the risky cash flow \((1 - \lambda)l\) to the household via the MMF in the shadow banking sector. Trading between the bank and the MMF in the shadow banking sector takes place as follows. To avoid trivial zero-trade equilibria and have a unique equilibrium,\(^8\) I suppose that the uninformed party, the MMF, offers a linear price \(r\) at which he is willing to buy stakes in the portfolio. The MMF is competitive: he quotes the highest possible price subject to breaking even (so that the household accepts financing him).

A bank with a good portfolio and no valuable opportunity turns down any price lower than one, and would be mimicked by a bad bank at this price. Therefore, there cannot be an equilibrium in which a good bank with no opportunity trades. The MMF thus quotes the price

\[
r = \frac{pq}{1 - p + pq},
\]

which is the probability that the bank is good and has a valuable opportunity conditionally on the bank trading. Such a good bank chooses to sell a stake \(\lambda'\) that maximizes its utility subject to being mimicked by a bad bank. Formally, it solves

\[
\max_{\lambda' \in [0, 1 - \lambda]} (p\lambda + r\lambda') l + L - I + f \left((p\lambda + r\lambda') l + L - I\right) + (1 - \lambda - \lambda')l. \tag{9}
\]

To characterize the equilibrium in the shadow banking sector, it is convenient to introduce the function \(\varphi\) defined as

\[
\varphi = f^{t-1},
\]

\(^8\)Equilibrium uniqueness is important because it pins down the beliefs of the regulator about shadow banking activity when he makes a date-0 decision on \(\lambda\).
a decreasing bijection from \((0, +\infty)\) into \((0, +\infty)\). Ignoring the constraint that \(\lambda' \in [0, 1 - \lambda]\), the first-order condition for program (9) reads:

\[(p\lambda + r\lambda') l = \varphi\left(\frac{1-p}{pq}\right) + I - L.\]  

(10)

The interaction between the bank and the MMF in the shadow banking sector resembles the stage game of Glosten and Milgrom (1985), where an uninformed competitive market maker (the MMF here) faces either an informed trader (the bad bank) or a liquidity trader (the good bank with a valuable opportunity). The only difference is that traded quantities are endogenous. Equation (10) formalizes the idea that tighter regulatory constraints (a smaller \(\lambda\)) spurs more shadow banking activity (a larger \(\lambda'\)). Let

\[\lambda = \frac{\varphi\left(\frac{1-p}{pq}\right) + I - L}{pl}, \quad \lambda' = \frac{\varphi\left(\frac{1-p}{pq}\right) + I - L - rl}{(p-r)l}.\]

**Lemma 4.** If \(\lambda \geq \lambda\), then the shadow banking sector is inactive (\(\lambda' = 0\)). If \(\lambda \leq \lambda\), then the bank sells its entire residual stake \((1 - \lambda)l\) to the MMF.

**Proof.** See the Appendix.

In words, the informational problems in the unregulated shadow banking sector are ex ante desirable in the sense that they make trading more costly to the bank. The threshold \(\lambda\) is particularly low when the bank is not very constrained (\(q\) small) and the expected quality of its portfolio is low (\(p\) small). Notice that the model predicts that regulatory arbitrage and shadow banking activity may arise even if bank capital regulation and the opportunity cost of bank equity remain constant. All that is needed is the perception of portfolio quality improvements, that is, that \(p\) increases holding \(pl\) constant. Optimistic views on the real estate market have trig-
gered such changes in the assessment of the risk to mortgages in the years preceding the crisis.

For brevity, the remainder of the paper restricts the analysis to the most interesting case in which
\begin{equation}
\lambda < 1.
\end{equation}

To make this parameter restriction compatible with the previous ones, I introduce an extra parameter by assuming that the entrepreneur internalizes only a fraction \( \alpha k \) of the total adjustments costs \( k \) that he generates, so that (2) becomes
\[ \alpha k \leq \min \{c; 1 - c\} \]

A straightforward microfoundation is that the entrepreneur purchases inputs from risk-averse suppliers or workers without commitment, thereby creating ex ante disutility costs for them when adjusting scale ex post. The regulator takes these agents into account when maximizing total surplus. It is easy to see that (11) implies (is, in fact, equivalent to):
\[ \bar{\lambda} > \hat{\lambda}. \]

Lemma 5. Suppose that \( \lambda_1, \lambda_2 \in [0, 1] \) satisfy
\[ \hat{\lambda} \leq \lambda_1 < \lambda_2 \leq \bar{\lambda}. \]

A regulatory leverage \( \lambda_2 \) strictly Pareto dominates a regulatory leverage \( \lambda_1 \). The utility of the bank is the same under both leverages whereas adjustment costs are strictly higher under \( \lambda_2 \).

Proof. See the Appendix. 

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This formalizes the broad idea that tightening capital requirements may be counterproductive in the presence of the shadow banking sector. Here, tighter capital requirements are highly counterproductive in the sense that they are Pareto dominated. As a result, the optimal regulatory leverage, if it is interior (different from 0 or 1), cannot be strictly between \( \lambda^\text{\hat{}} \) and \( \lambda^\text{\bar{}} \).

The intuition is simple. Condition (10) states that the date-1 resources of a bank with a valuable alternative investment do not depend on \( \lambda \) as \( \lambda \) varies within \([\lambda^\text{\hat{}}, \lambda^\text{\bar{}}]\). The bank undoes any regulatory restriction in the shadow banking sector. In response to a reduction \(-d\lambda \) in regulatory leverage, however, shadow leverage increases by

\[
d\lambda' = \frac{p}{r} d\lambda > d\lambda.
\]

That is, the lemons premium in the shadow sector leads the bank to transfer more risk per dollar raised in the shadow sector than with regulated deposits.

Lemmas 4 and 5 suggest conflicting effects of adverse selection in the shadow sector. On one hand, Lemma 4 implies that adverse selection curbs shadow banking by reducing the amount of cash that the bank seeks to raise in this sector. On the other hand, Lemma 5 shows that adverse selection increases the quantity of risk transfer per dollar raised. Here, I study these two effects for an arbitrary regulatory leverage \( \lambda \). The relevant impact of adverse selection, however, is that for an optimal leverage chosen ex ante by the regulator, which I solve for now.

Before doing so, it is worthwhile to make the following technical remark. The assumption that the MMF offers linear contracts is without loss of generality. If the MMF was competing with general contracts, separation could not be an equilibrium outcome. This is because in a separating equilibrium,
a bad bank would always take the contract that generates the highest date-1 payment. Thus, the good bank would take the contract with a lower payment, and would get less than the pooling price for its stake so that the MMF breaks even over all types. Deviating with a pooling contract attracts the good bank, and pooling is therefore the only sustainable competitive outcome if any. As is well known, the existence of an equilibrium in such competitive screening games depends on the exact equilibrium concept.

4.2.2 Optimal regulatory leverage in the presence of shadow banking

Proposition 3. a. If

\[ kpl\lambda \leq 1, \]

then the shadow banking sector is inactive (\( \lambda' = 0 \)), and the optimal capital requirement is \( \lambda^* \) as in Proposition 2: imperfect enforcement does not affect the equilibrium.

b. If

\[ kpl\hat{\lambda} \leq 1 < kpl\lambda, \]

then the shadow banking sector is inactive (\( \lambda' = 0 \)), and the optimal capital requirement is \( \lambda > \lambda^* \). Imperfect enforcement makes the bank better off but reduces utilitarian welfare.

c. If

\[ kpl\hat{\lambda} > 1, \]

there are two local maxima for total surplus, \( \lambda > \lambda^* \) and \( \lambda < \hat{\lambda}, \lambda^* \). The shadow banking sector is inactive at the highest leverage \( \lambda \), is fully active (\( \lambda' = 1 - \lambda \)) at the lowest \( \lambda \). In both cases, the bank is better off than under perfect enforcement, and total surplus is lower. Either local maximum can
be global, depending on parameter values.

**Proof.** See the Appendix. ■

The various regimes in Proposition 3 are spanned by, for example, letting the adjustment cost parameter $k$ vary, holding other parameters fixed. In case a, $k$ is sufficiently small, ceteris paribus, so that the optimal regulatory leverage under perfect enforcement $\lambda^*$ is large. The bank is not sufficiently constrained by regulation so that it feels the need to incur the trading costs associated with an opaque shadow sector. Shadow banking is irrelevant in this case.

In case b with a larger $k$, it is no longer so. Shadow banking is a relevant threat. The regulator does not handle this threat with tighter capital requirements. On the contrary, he relaxes the capital requirement ($\bar{\lambda} > \lambda^*$) up to the point at which the bank will not find any further refinancing in the shadow sector worthwhile. In other words, the regulator does himself at date 0 what the bank would do anyway at date 1—increase its effective leverage—in a socially more efficient fashion given that shadow banking transfers more risk to the household per dollar raised. As a result, there is no equilibrium shadow banking activity ($\lambda' = 0$). Yet the threat of shadow banking is effective as the bank is strictly better off with a higher feasible leverage $\bar{\lambda}$ than in the baseline model with perfect enforcement. Total surplus would be higher absent shadow banking under the smaller leverage $\lambda^*$.

Finally, in case c in which $k$ is the largest, there are two locally optimal capital requirements. First, the regulatory leverage $\bar{\lambda}$ that is optimal in case b is still locally optimal, and may or may not be the global optimum. Second, there exists another local optimum $\underline{\lambda} < \lambda^*$, whereby the strategy of the regulator is the polar opposite. Here, shadow banking is as active as it
can get ($\lambda' = 1 - \lambda$). Figure 2 depicts the two local maxima in this case c:

Figure 2 here.

If $q$ is sufficiently large (small) holding $qf$ constant, then $\lambda (\Delta)$ is the global optimum. The broad intuition is that if it is sufficiently highly likely that the bank will not be active in the shadow sector, then imposing a tight leverage that fully unleashes shadow banking with only a small probability is preferable to a high leverage.

4.2.3 The ex ante impact of adverse selection in equilibrium. For an arbitrary regulatory leverage $\lambda$, adverse selection in the shadow-banking sector has an unclear ex ante impact because it reduces the amount of cash raised by the bank, but increases risk transfer per dollar raised. I now study the impact of adverse selection at the equilibrium regulatory leverage determined in Proposition 3. Adverse selection in the shadow sector is larger when the private-value motive to trade is smaller - $q$ smaller so that a bank is less likely to trade because it has a good opportunity, and when the common-value motive to trade is larger - $p$ smaller so that the bank is more likely to try and sell a lemon. I am interested in how $p$, $q$ affect social surplus via their impact on shadow banking activity. Thus, the proper comparative-statics analysis is that in which changes in $p$, $q$ affect social surplus when imperfect enforcement makes shadow banking possible, while leaving social surplus unchanged in the case of perfect enforcement studied in Section 3, in which shadow banking is not allowed.

To carry out such an analysis, I introduce compensated changes in $(p, q)$ as follows. A compensated change in $q$ is a shift in parameters from $(q, f)$ to $(q', \frac{q}{q'}f)$. This way the probability that a bank receives a good opportunity
varies, while the ex ante expected return on the alternative investment does not, so that surplus is unaffected by changes in \( q' \) under perfect enforcement. In particular, the optimal regulatory leverage under perfect enforcement \( \lambda^* \) is unchanged. Similarly, a compensated change in \( p \) is a shift in parameters from \((p, k, l)\) to \((p', k', l')\) such that

\[
pl = p'l',
\]

\[
k(1-p)l = k'(1-p')l'.
\]

This way the probability that the portfolio is of high quality varies, while the expected payoff and the expected adjustment costs under perfect enforcement do not. In particular \( \lambda^* \) is unchanged.

**Proposition 4.** A compensated reduction in either \( p \) or \( q \) (higher adverse selection) leads to an increase in total surplus under imperfect enforcement whereas, by construction, it does not affect surplus under perfect enforcement.

**Proof.** See the Appendix. ■

The broad intuition for this result is the following. The negative consequences of adverse selection in the shadow sector—a reduction in price that increases risk transfer per dollar raised—is not important in equilibrium precisely because the regulatory leverage is chosen such that it does not matter ex post. Decreases in \( p \) and \( q \) are perhaps best interpreted as switches from “good times” to “bad times,” when expectations about bank assets quality are more pessimistic, and returns on alternative investment opportunities are more dispersed. Thus, the model predicts that the social costs of regulatory arbitrage in the shadow banking sector are more important in “good times.”
5. Discussion and Extensions

5.1 Inefficient regulation and efficient shadow banking

Thus far I have studied the benchmark of a benevolent rational planner who sets a constrained efficient capital requirement. It is not surprising that shadow banking can only lead to a reduction in social surplus in this case. It is interesting to study shadow banking in the alternative situation in which

$$\lambda^* \geq \lambda,$$

but in which the regulatory leverage is $\lambda < \lambda^$. That is, this corresponds to the situation in which the efficient regulation is sufficiently loose so that it will not trigger any shadow banking activity, whereas the implemented one is too tight and spurs shadow banking. This may stem from, for example, the fact that prudential regulation cannot be fully contingent on economic conditions, and is therefore too tight in good times.

In this context, one may wonder whether social surplus is larger under imperfect enforcement than if shadow banking is prohibited. I study this question using compensated comparative statics in $q$. That is, I let $q$ vary but suppose that the return function is of the form $q^{\frac{\theta}{q}} f$, where $q_0$ is fixed. Thus changes in $q$ do not affect the equilibrium under perfect enforcement.

**Proposition 5.** If $q$ is sufficiently close to one (low adverse selection), then surplus is higher if enforcement is imperfect than if it is perfect. If $q$ is sufficiently close to zero (high adverse selection), then the opposite holds true.

**Proof.** See the Appendix. ■

The intuition is that the shadow banking sector is desirable in the sense
that it helps the bank sidestepping an inefficiently tight capital constraint, but that it does so inefficiently by transferring more risk than necessary per dollar raised because of adverse selection. Which effect dominates depends on the magnitude of adverse selection.

It is interesting to contrast this result with that in Proposition 4. When regulatory leverage is optimally chosen, then adverse selection is desirable because it reduces the volume of shadow activity. When it is not, adverse selection reduces social surplus because the shadow banking sector restores a more efficient effective leverage ratio at the cost of important risk transfers. As a result, if one believes that the rise of the shadow banking sector is motivated by inappropriate capital regulations, then one should want informational problems in this sector to be as small as possible, for example, through reforms of OTC markets. If one believes, on the contrary, that shadow banking is meant to sidestep a broadly efficient prudential regulation, then one should prefer to maintain some opaqueness.

Finally, the result that unleashing shadow banking may be desirable when capital requirements are perceived as too tight may help understand the 2004 decision by U.S. regulators to set the deconsolidation of ABCP conduits mentioned above as a permanent exemption.

5.2 Alternative sources of illiquidity in the shadow banking sector

Adverse selection is a natural source of date-1 illiquidity in a shadow sector that refines information-problematic assets in opaque and unregulated OTC markets. In this particular setting, with only two bank types, adverse selection costs are linear. The reader may worry that this linearity drives the key result that endogenous increases in shadow leverage more than offsets
decreases in regulatory leverage. This may no longer be true with strictly convex transaction costs in the shadow sector. To assess this, suppose now that there is no adverse selection at date 1: the bank does not know anything about the portfolio that the MMF does not know. However, there are costs associated with secret trading, so that the sale of a claim \( \lambda' l \) in the shadow banking market generates only

\[
p l \int_0^{\lambda'} \delta(x) \, dx,
\]

where \( \delta \leq 1 \) is a decreasing function. In other words, there are convex costs from trading in the shadow banking sector.

Assuming \( q = l = 1 \) and \( L = I \) for notational simplicity, the bank now chooses to sell a fraction \( \lambda' \) of its portfolio so as to solve:

\[
\max_{\lambda' \in [0, 1 - \lambda]} \left\{ p(1 - \lambda - \lambda') + p\left( \lambda + \int_0^{\lambda'} \delta \right) + f \left( p\left( \lambda + \int_0^{\lambda'} \delta \right) \right) \right\}.
\]

Ignoring the constraint \( \lambda' \in [0, 1 - \lambda] \), the first-order condition with respect to \( \lambda' \) is:

\[
1 + f' \left( p\left( \lambda + \int_0^{\lambda'} \delta \right) \right) = \frac{1}{\delta(\lambda')}.
\]

Risk transfer in the shadow banking sector more than offsets the risk reduction due to tighter capital requirements when

\[
\frac{d\lambda'}{d\lambda} \leq -1.
\]
From (15),
\[
\frac{d\lambda'}{d\lambda} \leq -1 \iff \frac{\delta' (\lambda')}{(\delta (\lambda'))^2} \geq pf'' \left( p \left( \lambda + \int_0^{\lambda'} \delta \right) \right) \left( 1 - \delta (\lambda') \right),
\]
which holds if, other things being equal, the absolute value of \( \delta' \) is sufficiently small and that of \( f'' \) sufficiently large. The increase in marginal trading costs must be sufficiently small relative to the decrease in marginal investment returns.

5.3 Hidden leverage and hidden risk: A theory of excessive asset encumbrance

In this setup, the bank transfers risk to the household in a concealed fashion through hidden leverage, but it cannot alter the risk profile of its portfolio. Risk-adjusted leverage increases in the presence of shadow banking only because leverage increases. An interesting extension is that in which risk-adjusted leverage also increases because the shadow banking sector induces an endogenous increase in the risk profile transferred to the household. I sketch a simple version of it here. Suppose the bank has two assets: a “safe” (but not risk-free) one and a “risky” one. The safe asset pays off \( L \) with prob. \( P \) and zero otherwise. In this latter case of default, the risky asset pays off zero as well, whereas it pays off \( l \) with prob. \( p \) conditionally on the safe asset performing. Suppose

\[
L = pl.
\]

Under perfect enforcement, it is clearly optimal that the bank issues a claim backed by the entire safe asset, and by part of the risky asset if it is
not enough, which I assume. This minimizes risk transfer for a given amount of cash raised. Under imperfect enforcement, the bank would then possibly refinance an additional tranche of the risky asset with the MMF.

Suppose now that the regulator, unlike the other agents, cannot tell apart the safe and the risky asset. Then the bank would prefer to swap the role of these assets. That is, it would prefer to sell its entire risky asset in the formal banking sector together with part of the safe one, and a bigger residual fraction of the safe one in the shadow banking sector. This is because adverse selection is not a problem in formal banking because the bank can pool over future types, whereas it is one in the shadow banking sector. The bank therefore prefers to save the least information-sensitive asset for shadow banking activity, where it matters. But then, not only does shadow banking increase leverage on assets, it also increases the risk profile transferred to the household because of asset substitution.

Such an asset substitution strongly relates to the current debate about excessive asset encumbrance or “run on collateral” (see, e.g., BIS, 2013). The concern is that banks may now use too much of their high-quality assets as collateral in their shadow banking refinancing activities, leaving too few high-quality assets unencumbered to back the unsecured claims (including deposits) in their general balance sheet.

It is interesting to note that this extension and the baseline model have different predictions regarding the quality of the assets that are refinanced in the shadow sector relative to the unconditional quality of the asset class. In the baseline model, the probability of default of a portfolio conditionally on being refinanced in the shadow sector is higher than the unconditional one \( r < p \). In this extension, on one hand, it is still the case that a bank that has no valuable alternative investment and a good portfolio does
not tap the shadow sector, resulting in a higher proportion of lemons in
the shadow sector. On the other hand, the assets that are refinanced in
the shadow sector have a lower conditional probability of default than the
average unconditional probability of default, because banks use their best
collateral in the shadow sector. If this latter effect dominates, then it may
be that the assets that are refinanced in the shadow sector have a lower
probability of default than those that are not.

6. Conclusion

This paper builds a theory of the prudential regulation of banks on the
premises that the shareholders of banks, unlike those of other firms, inter-
nalize only a fraction of the social cost of default on their liabilities. This in
turn stems from the specific role of bank liabilities as media of exchange.

I first write a model of bank regulation with perfect enforcement that
formalizes this idea in a tractable and simple fashion. I then introduce
imperfect enforcement in this framework by simply assuming that the reg-
ulator cannot monitor all the markets in which bank assets are refinanced
with money-like liabilities. The small, but important, difference between the
regulated markets and the unregulated ones is that banks can commit to a
financing policy before they receive private information about their assets
in the former, whereas they trade ex post optimally once informed in the
latter.

As a result, shadow banking is plagued by adverse selection and formal
banking is not. The model yields clear-cut predictions on the ex ante impact
of the informational frictions in the shadow banking sector. If regulation is
set optimally, then adverse selection is desirable because it acts as a com-
mitment device for banks to have a more limited shadow banking activity. The optimal leverage in the presence of shadow banking is closer to that under perfect enforcement if adverse selection is more severe. If regulation is inefficiently tight, then adverse selection is costly because it induces banks to offload more risk than necessary in the shadow banking sector to bypass excessive capital requirements.

Appendix A

A.1 Proof of Lemma 1

Suppose that the household has a realized date-2 wealth $W_2$ and the entrepreneur made an initial capacity choice $N_1$. Without loss of generality, one can restrict the analysis to subgames in which

$$-l \leq W_2 - N_1 \leq l.$$ 

This is because it cannot be optimal that the household bears more risk than the total risk of the portfolio. It also cannot be optimal for the entrepreneur to adjust capacity almost surely in one direction.

Suppose that $W_2 \geq N_1$. Then the entrepreneur chooses $N_2 \in [N_1, W_2]$ so as to maximize

$$N_2 (1 - c) - \frac{k (N_2 - N_1)^2}{2}.$$ 

This is because given a capacity $N_2$, it is clearly optimal to set the unit price to one. Condition (2) implies that $N_2 = W_2$ is optimal.

Suppose now that $W_2 < N_1$. The entrepreneur now chooses $N_2 \in$
so as to maximize
\[ W_2 - N_2 c - \frac{k(N_2 - N_1)^2}{2}, \]

because it is optimal to charge a unit price \( \frac{W_2}{N_2} \) in this case. Condition (2) implies that \( N_2 = W_2 \) is optimal. Here, we implicitly assume that \( W \) is sufficiently large so that the entrepreneur’s profit is always positive.

**A.2 Proof of Lemma 2**

From Lemma 1, the entrepreneur chooses \( N_1 \) so as to maximize
\[ (1 - c) E(W_2) - \frac{kE[(W_2 - N_1)^2]}{2}. \]

It is therefore optimal to set
\[ N_1 = E(W_2). \]

**A.3 Proof of Proposition 1**

Inspection of (7) shows that \( \mu^* = 1 \) is optimal, and condition (8) stems from the first-order condition w.r.t. \( \lambda \) in (7). Condition (3) implies that the solution to (8) is smaller than one. Because \( \lambda^*pl + L - I \) must be strictly positive from the Inada condition at zero, it must be that the loan can be financed when leverage is \( \lambda^*).
A.4 Proof of Proposition 2

First, it is easy to see that it is optimal that a bank always pledges its entire risk-free cash flow $L$ at date 1. The design of the optimal security therefore boils down to determining the fraction of the risky cash flow $l$ that each date-1 type of bank sells to the household at date 1. I denote $P$ as the random variable that describes the net payoff to the household for a given mechanism and $Q \in \{G; B\}$ as the type of the portfolio. The total variance formula viewed from date 0 writes

$$Var(P) = E(Var(P \mid Q)) + Var(E(P \mid Q)).$$

Suppose first that all types pool at date 1. For some fixed $\lambda \in [0, 1]$, all types issue a claim $\lambda l$ against their risky cash flow sold at the price $\lambda pl$. The net payoff to the household is

$$\begin{cases} 
-\lambda pl & \text{if } Q = B \\
(1 - \lambda) pl & \text{if } Q = G
\end{cases},$$

so that

$$E(Var(P \mid Q)) = Var(P \mid G) = Var(P \mid B) = 0,$$

$$Var(E(P \mid Q)) = Var(P) = p(1 - p)\lambda^2 L^2.$$

I now show that any other mechanism such that a bank with a good portfolio and a valuable investment opportunity raises at least $\lambda pl$ implies a larger variance $Var(P)$. Suppose that a mechanism is such that a bank with a good portfolio and no valuable alternative investment raises an amount of cash $\pi_n$
at date 1 against a claim to \( \lambda_n l \), whereas a bank with a good portfolio and a valuable investment opportunity raises \( \pi_i \geq \lambda pl \) against a claim \( \lambda l \). The key remark is that a bank with a bad portfolio always mimics the contract that raises the largest amount. Thus, the payoff to the household if the portfolio is bad is deterministic, equal to

\[
- \max (\pi_i; \pi_n) \leq -\lambda pl.
\]

Because the household accepts each security \((i, n)\) only if he breaks even for each of them, it must be that he breaks even ex ante, so that

\[
E(P \mid G) \geq (1 - p)\lambda l.
\]

This implies

\[
Var(E(P \mid Q)) \geq p(1 - p)\lambda^2 L^2.
\]

Because in addition

\[
E(Var(P \mid Q)) \geq 0
\]

for any set of securities, this establishes the result.

A.5 Proof of Lemma 4

An inspection of (10) shows that constraint \( \lambda' \geq 0 \) binds for \( \lambda \geq \bar{\lambda} \) and so does constraint \( \lambda' \leq 1 - \lambda \) for \( \lambda \leq \hat{\lambda} \).
A.6 Proof of Lemma 5

It is easy to see that the respective date-0 utilities of the bank and the entrepreneur in the presence of shadow banking are

\[
U_B = pl + L - I + qf \left( (\lambda p + \lambda' r) l + L - I \right), \\
U_E = W(1 - c) - \frac{k (1 - p) l^2}{2} \left[ p\lambda^2 + r\lambda'^2 + 2r\lambda' \right].
\]

For \( \lambda \in \left[ \hat{\lambda}, \bar{\lambda} \right] \), condition (10) implies that these utilities become

\[
U_B = pl + L - I + qf \left( \frac{1 - p}{pq} \right), \\
U_E = W(1 - c) - \frac{k (1 - p)}{2p} \left[ \left( \frac{1 - p}{pq} \right) + I - L \right]^2 + \frac{p - r}{r} \left( \frac{1 - p}{pq} + I - L - \lambda pl \right)^2,
\]

and the result follows because \( \lambda \leq \bar{\lambda} \) implies that \( \varphi \left( \frac{1 - p}{pq} \right) + I - L - \lambda pl \) is positive.\( \blacksquare \)

A.7 Proof of Proposition 3

a. Condition (12) means that \( \lambda^* \geq \bar{\lambda} \). This implies that the shadow banking market is inactive at \( \lambda^* \), which therefore still is the optimal leverage.

b. Utilities (16) and (16) imply that total surplus varies as follows when \( \lambda \) describes \( \left[ 0, \hat{\lambda} \right] \) (notice that this set may be empty). It admits an interior maximum \( \lambda^* \), which solves

\[
qf' \left( (\lambda (p - r) + r) l + L - I \right) = \lambda k (1 - p) L
\]

if \( pk\bar{\lambda} > 1 \) and is maximal at \( \hat{\lambda} \) otherwise.
Condition (13) thus implies that social surplus is increasing w.r.t. \( \lambda \) over \([0, \hat{\lambda}]\). It also implies that \( \lambda^* < \overline{\lambda} \), so that social surplus decreases w.r.t. \( \lambda \) over \([\overline{\lambda}, 1]\). Lemma 5 implies that surplus also increases w.r.t. \( \lambda \) over \([\hat{\lambda}, \overline{\lambda}]\). Thus, surplus is maximal for leverage \( \overline{\lambda} \). Shadow banking is inactive but is a threat that benefits the bank and overall reduces surplus because \( \overline{\lambda} > \lambda^* \).

c. Otherwise, if \( pk\hat{\lambda} > 1 \), then \( \Lambda \) as defined above in (6) also corresponds to a locally maximum total surplus together with \( \overline{\lambda} \). To see that \( \Lambda < \lambda^* \), notice that for all \( \lambda < 1 \),

\[
\lambda(p - r) + r > \lambda p.
\]

Thus it must be that \( \Lambda \), which solves (6), and \( \lambda^* \), which solves (8), satisfy:

\[
\Lambda < \lambda^*, \quad \Lambda(p - r) + r > \lambda^* p,
\]

this latter inequality means that the bank is better off under \( \Lambda \) than under \( \lambda^* \).

Finally, I show that either local optimum can be global. Notice that as \( q \) varies, while \( f \) is scaled so that \( qf \) remains constant, then \( \Lambda \) and \( \hat{\lambda} \) vary, while \( \lambda^* \) and \( \overline{\lambda} \) do not. In particular, as \( q \to 0 \), then \( \Lambda \to \lambda^* \) from (6), and \( \Lambda \) is therefore the global optimum. When \( q \) increases so that \( pk\hat{\lambda} \downarrow 1 \), then \( \Lambda \uparrow \overline{\lambda} \), and \( \overline{\lambda} \) is the global optimum from Lemma 5. ■

**A.8 Proof of Proposition 4**

Suppose that the optimal leverage is \( \lambda^* \geq \overline{\lambda} \). A compensated reduction in \( q \) leaves \( \overline{\lambda} \) unchanged, whereas a compensated reduction in \( p \) reduces it, so
that optimal leverage is still $\lambda^*$, and social surplus is not affected by such compensated variations in $p$, $q$.

Suppose that the optimal leverage is $\bar{\lambda}$. Then surplus is, up to constant terms,

$$qf(\bar{\lambda} pl + L - I) - \frac{k}{2} p(1 - p) \lambda^2.$$  
Again, a compensated reduction in $q$ does not affect surplus. A compensated reduction in $p$ reduces $\bar{\lambda}$, which increases surplus because $\bar{\lambda} > \lambda^*$. Perhaps surplus becomes even higher by choosing the other local maximum with full shadow banking activity.

Suppose that the optimal leverage is $\lambda$. Then surplus, up to constant terms, is

$$qf\left((\lambda \left(1 - \frac{r}{p}\right) + \frac{r}{p}) pl + L - I\right) - \frac{k}{2} (1 - p) pl^2 \left((1 - \frac{r}{p}) \lambda^2 + \frac{r}{p}\right), \quad (A.1)$$

with

$$qf'\left((\lambda \left(1 - \frac{r}{p}\right) + \frac{r}{p}) pl + L - I\right) = k(1 - p)l\lambda. \quad (A.2)$$

Differentiating (A.1) w.r.t. $\frac{r}{p}$ using the envelope theorem yields

$$qf'\left((\lambda \left(1 - \frac{r}{p}\right) + \frac{r}{p}) pl + L - I\right) (1 - \lambda) - k(1 - p)l(1 - \lambda^2),$$

which is negative from (A.2). Further,

$$\frac{r}{p} = \frac{q}{1 - p + pq}$$

increases w.r.t. $q$, $p$. Perhaps, surplus becomes even higher by choosing the other local maximum without active shadow banking. ■
A.9 Proof of Proposition 5

The respective social surpluses under perfect and imperfect enforcement are:

\[ S_{\text{perfect}} = q_0 f(\lambda pl + L - I) - \frac{k}{2} p(1 - p) l^2 \lambda^2, \]
\[ S_{\text{imperfect}} = q_0 f(\bar{\lambda} pl + L - I) - \frac{k}{2} p(1 - p) l^2 \left( \bar{\lambda}^2 + \frac{p - r}{r} (\bar{\lambda} - \lambda)^2 \right). \]

Denoting
\[ \Delta (\lambda) = S_{\text{perfect}} - S_{\text{imperfect}}, \]
one has \( \Delta (\bar{\lambda}) = 0 \), and

\[ \Delta' (\lambda) = pl \left[ \underbrace{-q_0 f' (\lambda pl + L - I)}_{A} + \underbrace{k (1 - p) l \lambda}_{A} + \underbrace{\frac{k (1 - p) l (p - r)}{r}}_{B} (\bar{\lambda} - \lambda) \right], \]

where \( A < 0 \) because leverage \( \lambda \) is strictly lower than \( \lambda^* \), and \( B > 0 \). The only term that \( q \) affects is

\[ \frac{p - r}{r} = \frac{(1 - p)(1 - q)}{q}, \]

which decreases from \(+\infty\) to 0 as \( q \) spans \((0, 1)\). Thus, for \( q \) sufficiently large (small), \( \Delta(\lambda) \leq (\geq) 0. \]
References


Figure 1. Bank balance sheet after initial cash transfers are made
Figure 2. Surplus in Proposition 3 case c
Online appendix: two simple extensions

Continuous payoff

The assumption that the loan portfolio payoff has a binary support \( \{ L; L + l \} \) enables us to simply characterize the optimal security sold to the household with a single number \( \lambda \). Consider an extension in which the portfolio pays off instead \( \tilde{R} \), where \( \tilde{R} \) is a random variable with a continuous c.d.f. \( F \) whose support is included in \([0, \tilde{R}]\) for some \( \tilde{R} > 0 \). In this case, a regulator maximizing utilitarian welfare now designs a security \( S(\cdot) \) such that the bank receives \( S(R) \) when the portfolio pays off \( R \). The household receives a random amount \( R - S(R) \) from his claim to the portfolio against a deposit \( E[R - S(R)] \), and invests the residual \( E[W - R + S(R)] \) with the MMF.

It is optimal that the bank retains equity and sells debt to the household:

\[
S(R) = \max (R - d; 0),
\]

where \( d \in [0, \tilde{R}] \) solves

\[
qf' \left( \int_0^d (1 - F) - I \right) = k \int_0^d F.
\]

**Proof.** The program facing the regulator is a mean-variance program. Solving it therefore boils down to finding a security \( S \) that solves

\[
\min_S Var (R - S(R))
\]

\[
s.t. \begin{cases} 
E(S(R)) = X, \\
\forall R \in [0, \tilde{R}],\ S(R) \geq 0, 
\end{cases}
\]

(1)
where \( X \) is a constant that belongs to \((0, E(R))\). The constraint \( S \geq 0 \) stems from the limited liability of the bank. Notice that I do not take into account the household’s limited-liability constraint: the solution to this program satisfies it. Let \( d \in [0, \bar{R}] \) s.t.

\[
E((R - d)^+) = X.
\]

For any given security \( S \) that satisfies constraints (1),

\[
Var(R - S(R)) = Var(R) + Var(S(R)) - 2\text{cov}(R, S(R)),
\]

and

\[
\text{cov}(R, S(R)) = \text{cov}(R - d, S(R))
\]

\[
\leq \text{cov}((R - d)^+, S(R)) + (E((R - d)^+) - E(R - d))E(S(R))
\]

\[
= \text{cov}((R - d)^+, S(R)) + E((d - R)^+)X.
\]

Thus, using Cauchy-Schwartz inequality,

\[
Var(R - S(R)) \geq Var(R) + Var(S(R)) - 2E((d - R)^+)X - 2\sqrt{Var((R - d)^+)Var(S(R))}.
\]

Letting \( t = \sqrt{\frac{VarS(R)}{Var((R - d)^+)}} \), I can rewrite (2) as

\[
Var(R - S(R)) \geq VarR - 2E((d - R)^+)X + (t^2 - 2t)Var((R - d)^+).
\]

The minimum of the right-hand side of (4) is reached at \( t = 1 \), and inequality (4) is an equality when \( S(R) = (R - d)^+ \), which yields the result that equity
is optimal. The expected payoff and variance of the complementary debt claim to the household are respectively

\[
\int_0^d (1 - F),
\int_0^d 2x(1 - F(x)) dx - \left( \int_0^d 1 - F \right)^2.
\]

The regulator sets \(d\) so as to maximize

\[
q_f \left( \int_0^d (1 - F) - I \right) - k \left( \int_0^d 2x(1 - F(x)) dx - \left( \int_0^d 1 - F \right)^2 \right),
\]

and the first-order condition on \(d\) yields

\[
q_f' \left( \int_0^d (1 - F) - I \right) = k \int_0^d F.
\]

\[\Box\]

The intuition is simply that the program facing the regulator is basically a mean-variance program. Equity is the feasible security (given a limited-liability condition \(S(R) \geq 0\) that minimizes the variance of the household claim for a given amount of cash raised by the bank against the loan portfolio. Clearly, the comparative statics properties of \(\lambda^*\) in the baseline model with binary payoff equally apply for \(d\) here.

**Deposit insurance**

While a commitment problem prevents non-bank agents from entering into private deposit-insurance arrangements, I could assume that the regulator has deep pockets and can provide public deposit insurance. If public insurance came at no cost, then optimally deposits would be fully insured,
and the bank fully leveraged. If, more plausibly, public insurance came at some (weakly convex) deadweight cost, then the regulator would in general implement a policy mix of capital requirements for banks and partial deposit insurance. The mix would be such that the marginal opportunity cost of bank capital, the marginal cost of deposit insurance, and the marginal adjustment costs due to residual deposit risk are equal. (It is possible to equate three marginal costs with two policy variables.) If, alternatively, the planner could not commit to such partial insurance, and instead fully bailed out depositors, then he would impose higher capital requirements than that when partial insurance is credible. Detailing these results is beyond the scope of this paper. I only stress that abstracting from deposit insurance and bail-outs is for expositional simplicity only, and not substantial. All that matters is that perfect insurance of bank deposits is not feasible. The reason assumptions about deposit insurance or bail-outs do not play a crucial role here is that I directly model the primitive market failures that motivate such interventions in the first place.